CHAPTER 6

ELECTROMAGNETIC INDUCTION

6.1 MAGNETIC FLUX

1. *Explain the concept of magnetic flux linked with a closed surface. Give its units and dimensions. When is the magnetic flux said to be (i) positive and (ii) negative ?*

Magnetic flux. *The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally.* It is measured as the product of the component of the magnetic field normal to the surface and the surface area.

Magnetic flux is a scalar quantity, denoted by ϕ or ϕ_B.

If a uniform magnetic field $\vec{B}$ passes normally through a plane surface area A, as shown in Fig. 6.1(*a*), then the magnetic flux through this area is

$$\phi = BA$$

If the field $\vec{B}$ makes angle θ with the normal drawn to the area A, as shown in Fig. 6.1(*b*), then the

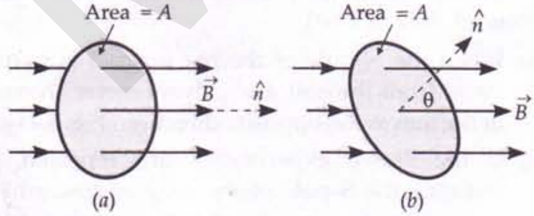


Fig. 6.1 Magnetic flux through an area depends on its orientation w.r.t. the magnetic field.

component of the field normal to this area will be $B\cos\theta$, so that

$$\phi = B\cos\theta \times A$$

or

$$\phi = BA\cos\theta = \vec{B}.\vec{A}$$

Here the direction of vector $\vec{A}$ is the direction of the outward drawn normal to the surface.

In general, the field $\vec{B}$ over an area A may not be uniform. However, over a small area element $d\vec{A}$, the field $\vec{B}$ may be assumed to be uniform. As shown in Fig. 6.2, if θ is the angle between $\vec{B}$ and the normal drawn to area element $d\vec{A}$, then the component of $\vec{B}$ normal to $d\vec{A}$ will be $B\cos\theta$.

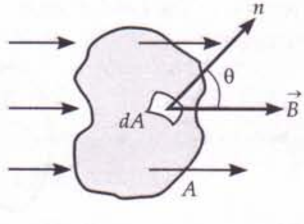


Fig. 6.2 Surface A in a magnetic field.

∴ Flux through area element $d\vec{A}$ is

$$d\phi = B_{\perp}\, dA = B\cos\theta\, dA$$
$$= B\,dA\cos\theta = \vec{B}\cdot d\vec{A}$$

Then the flux of $\vec{B}$ through the whole area A is

$$\phi = \int_A \vec{B}\cdot d\vec{A}$$

Dimensions of magnetic flux. As we know that

$$\phi = BA$$

But $B = \frac{F}{qv\sin\theta}$ ∴ $\phi = \frac{F}{qv\sin\theta}\cdot A$

Dimensions of flux,

$$\phi = \frac{\text{MLT}^{-2}}{\text{C}\cdot\text{LT}^{-1}\cdot 1}\cdot\text{L}^2$$
$$= \frac{\text{ML}^2\text{T}^{-2}}{\text{A}} \qquad [\because 1\,\text{CT}^{-1} = 1\,\text{A}]$$

or $[\phi] = [\mathbf{ML^2A^{-1}T^{-2}}]$.

SI unit of magnetic flux. The SI unit of magnetic flux is ***weber*** **(Wb)**. *One weber is the flux produced when a uniform magnetic field of one tesla acts normally over an area of 1 m².*

$$1\text{ weber} = 1\text{ tesla}\times 1\text{ metre}^2$$

or $1\text{ Wb} = 1\text{ Tm}^2$

CGS unit of magnetic flux. The CGS unit of magnetic flux is ***maxwell* (Mx)**. *One maxwell is the flux produced when a uniform magnetic field of one gauss acts normally over an area of 1 cm².*

$$1\text{ maxwell} = 1\text{ gauss}\times 1\text{ cm}^2$$

or $1\text{ Mx} = 1\text{ G cm}^2$

Relation between weber and maxwell

$$1\text{ Wb} = 1\text{ T}\times 1\text{ m}^2 = 10^4\text{ G}\times 10^4\text{ cm}^2$$

or $1\text{ Wb} = 10^8$ maxwell.

Positive and negative flux. A normal to a plane can be drawn from either side. If the normal drawn to a plane points out in the direction of the field, then $\theta = 0°$ and the flux is taken as *positive*. If the normal points in the opposite direction of the field, then $\theta = 180°$ and the flux is taken as *negative*.

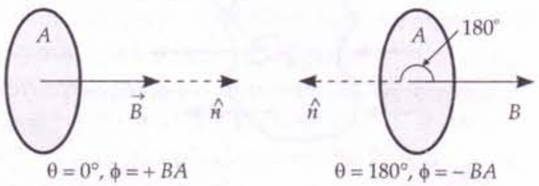


Fig. 6.3 (*a*) Positive flux and (*b*) Negative flux.

6.2 ELECTROMAGNETIC INDUCTION : AN INTRODUCTION

2. *What is electromagnetic induction ?*

Electromagnetic induction. Electricity and magnetism are intimately connected. In the early part of the nineteenth century, the experiments of Oersted, Ampere and others established that moving charges (currents) produce a magnetic field. The converse effect is also true *i.e.*, moving magnets can produce electric currents. In 1831, *Michael Faraday* in England and almost simultaneously *Joseph Henry* in the U.S.A. discovered that currents are produced in a loop of wire if a magnet is suddenly moved towards the loop or away from the loop such that the magnetic flux across the loop changes. The current in the loop lasts so long as the flux is changing. This phenomenon is called ***electromagnetic induction*** which means *inducing electricity by magnetism.*

The phenomenon of production of induced emf (and hence induced current) due to a change of magnetic flux linked with a closed circuit is called electromagnetic induction.

The phenomenon of electromagnetic induction is of great practical importance in daily life. It forms the basis of the present day generators and transformers. Modern civilisation owes a great deal to the discovery of electromagnetic induction.

6.3 FARADAY'S EXPERIMENTS

3. *Describe the various experiments performed by Faraday and Henry which ultimately led to the discovery of the phenomenon of electromagnetic induction.*

Faraday's experiments. The phenomenon of electromagnetic induction was discovered and understood on the basis of the following experiments performed by Faraday and Henry.

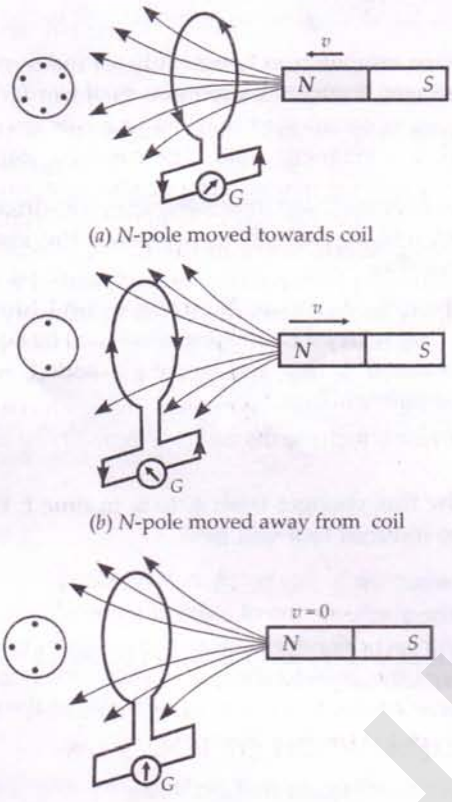
Experiment 1. Induced emf with a stationary coil and moving magnet. As shown in Fig. 6.4, take a circular coil of thick insulated copper wire connected to a sensitive galvanometer.

(*i*) When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection, say to the right of the zero mark [Fig. 6.4 (*a*)].

(*ii*) When the N-pole of the bar magnet is moved away from the coil, the galvanometer shows a deflection in the opposite direction [Fig. 6.4 (*b*)].

(*iii*) If the above experiments are repeated by bringing the S-pole of the magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.

(*iv*) When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection [Fig. 6.4 (*c*)].



(*a*) *N*-pole moved towards coil

(*b*) *N*-pole moved away from coil

(*c*) Magnet at rest.

Fig. 6.4 Induced emf with a moving magnet and stationary coil.

Explanation. When a bar magnet is placed near a coil, a number of lines of force pass through it. As the magnet is moved closer to the coil, the magnetic flux (the total number of magnetic lines of force) linked with the coil increases, an induced emf and hence an induced current is set up in the coil in one direction. As the magnet is moved away from the coil, the magnetic flux linked with the coil decreases, an induced emf and hence an induced current is set up in the coil in the opposite direction. As soon as the relative motion between the magnet and the coil ceases, the magnetic flux linked with the coil stops changing and so the induced current through the coil becomes zero.

Experiment 2. Induced emf with a stationary magnet and moving coil. Similar results as in experiment 1 are obtained if the magnet is held stationary and the coil is moved, as shown in Fig. 6.5. When the relative motion between the coil and the magnet is fast, the deflection in the galvanometer is large and when the relative motion is slow, the galvanometer deflection is small.

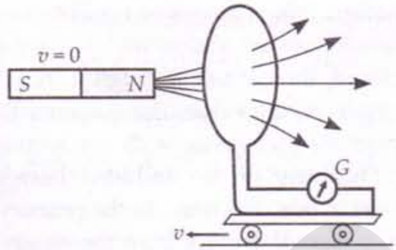


Fig. 6.5 Electromagnetic induction with a stationary magnet and moving coil.

Faster the relative motion between the magnet and the coil, greater is the rate of change of magnetic flux linked with the coil and larger is the induced current set up in the coil.

Experiment 3. Induced emf by varying current in the neighbouring coil. Fig. 6.6 shows two coils *P* and *S* wound independently on a cylindrical support. The coil *P*, called *primary coil*, is connected to a battery and a rheostat through a tapping key *K*. The coil *S*, called *secondary coil*, is connected to a sensitive galvanometer.

(*i*) When the tapping key is pressed, the galvanometer shows a momentary deflection in one direction (Fig. 6.6). When the key is released, it again shows a momentary deflection but in the opposite direction.

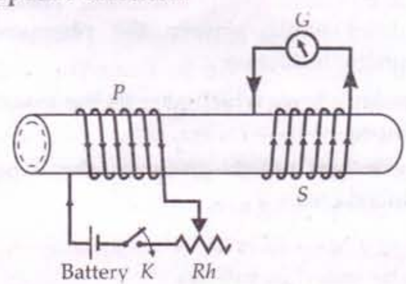


Fig. 6.6 Electromagnetic induction by varying current in the neighbouring coil.

(*ii*) If the tapping key is kept pressed and steady current flows through the primary coil, the galvanometer does not show any deflection.

(*iii*) As the current in the primary coil is increased with the help of the rheostat, the induced current flows in the secondary in the same direction as that at the *make* of the primary circuit.

(*iv*) As the current in the primary coil is decreased, the induced current flows in the same direction as that at the *break* of the primary circuit.

(*v*) The deflections in the galvanometer become larger if we use a cylindrical support made of iron.

Explanation. When a current flows through a coil, a magnetic field gets associated with it. As the primary circuit is closed, the current through it increases from zero to a certain steady value. The magnetic flux linked with the primary and hence with the secondary also increases. This sets up an induced current in the secondary coil in one direction. As the primary circuit is broken, the current decreases from the steady value to zero, the magnetic flux through the secondary coil decreases. An induced current is set up in the secondary coil but in the opposite direction. When a steady current flows in the primary coil, the magnetic flux linked with the primary coil does not change and no current is induced in the secondary coil.

From these experiments, we may conclude that :

1. *Whenever the magnetic flux linked with a closed circuit changes, an induced emf and hence an induced current is set up in it.*
2. *The higher the rate of change of magnetic flux linked with the closed circuit, the greater is the induced emf or current.*

6.4 LAWS OF ELECTROMAGNETIC INDUCTION

4. *State the laws of electromagnetic induction. Express these laws mathematically.*

Laws of electromagnetic induction. There are two types of laws which govern the phenomenon of electromagnetic induction :

A. Faraday's laws which give us the magnitude of induced emf.

B. Lenz's law which gives us the direction of induced emf.

A. Faraday's laws of electromagnetic induction : These can be stated as follows :

First law. *Whenever the magnetic flux linked with a closed circuit changes, an emf (and hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomenon is called electromagnetic induction.*

Second law. *The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit. Mathematically,*

$$|\varepsilon| = \frac{d\phi}{dt}$$

B. Lenz's law : *This law states that the direction of induced current is such that it opposes the cause which produces it, i.e., it opposes the change in magnetic flux.*

Mathematical form of the laws of electromagnetic induction : Expression for induced emf. According to the Faraday's flux rule,

Magnitude of induced emf

= Rate of change of magnetic flux

or $$|\varepsilon| = \frac{d\phi}{dt}$$

Taking into account Lenz's rule for the direction of induced emf, Faraday's law takes the form :

$$\varepsilon = -\frac{d\phi}{dt}$$

The negative sign indicates that the direction of induced emf is such that it opposes the change in magnetic flux.

If the coil consists of *N* tightly wound turns, then the emfs developed in all these turns will be equal and in the same direction and hence get added up. Total induced emf will be

$$\varepsilon = -N\frac{d\phi}{dt}$$

If the flux changes from ϕ_1 to ϕ_2 in time *t*, then the average induced emf will be

$$\varepsilon = -N\frac{\phi_2 - \phi_1}{t}$$

If ϕ is in webers and *t* in seconds, then ε will be in volts.

6.5 EXPLANATION OF LENZ'S LAW

5. *State and illustrate Lenz's law.*

Lenz's law. In 1833, German physicist *Heinrich Lenz* gave a general law for determining the direction of induced emf and hence that of induced current in a circuit.

Lenz's law states that the direction of induced current in a circuit is such that it opposes the cause or the change which produces it.

Thus, if the magnetic flux linked with a closed circuit increases, the induced current flows in such a direction so as to create a magnetic flux in the opposite direction of the original magnetic flux. If the magnetic flux linked with the closed circuit decreases, the induced current flows in such a direction so as to create a magnetic flux in the direction of the original flux.

Illustrations of Lenz's law :

(*i*) When the north pole of a bar magnet is moved towards a closed coil, the induced current in the coil flows in the anticlockwise direction, as seen from the magnet side [Fig. 6.7(*a*)] closely. The face of the coil towards the magnet develops north polarity and thus, it opposes the motion of the north pole of the magnet towards the coil which is actually the cause of the induced current in the coil.

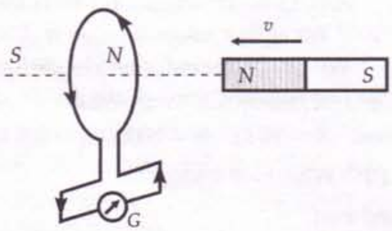


Fig. 6.7 (*a*) Direction of induced current when a magnet moves towards a coil.

In other words, the motion of the magnet increases the flux through the coil. The induced current generates flux in opposite direction, and hence opposes and reduces this flux.

(*ii*) When the north pole of a magnet is taken away from a closed coil, the induced current in the coil flows clockwise, as seen from the magnet side. The face of the coil towards the magnet develops south polarity and attracts the north pole of the magnet, *i.e.*, the motion of the magnet away from the coil is opposed which is really the cause of the induced current [Fig. 6.7(*b*)].

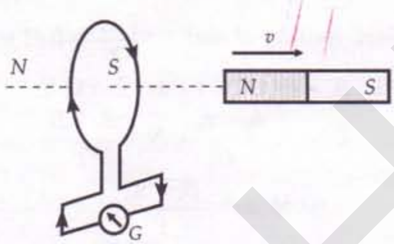


Fig. 6.7 (*b*) Direction of induced current when a magnet moves away from a coil.

In other words, the motion of the magnet decreases the flux through the coil. The induced current generates flux in the same direction and hence opposes and increases this flux.

6. *Show that Lenz's law is a consequence of the law of conservation of energy.*

Lenz's law and law of conservation of energy. Whether a magnet is moved towards or away from a closed coil, the induced current always opposes the motion of the magnet, as predicted by Lenz's law. For example, when the north pole of a magnet is brought closer to a coil [Fig. 6.7(*a*)], its face towards the magnet develops north polarity and thus repels north pole of the magnet. Work has to be done in moving the magnet closer to the coil against this force of repulsion. Similarly, when the north pole of the magnet is moved away from the coil [Fig. 6.7(*b*)], its face towards the magnet develops south polarity and thus attracts the north pole of the magnet. Here work has to be done in moving the magnet away from the coil against this force of attraction. It is this work done against the force of repulsion or attraction that appears as electric energy in the form of induced current.

Suppose that the Lenz's law is not valid. Then the induced current flows through the coil in a direction opposite to one dictated by Lenz's law. The resulting force on the magnet makes it move faster and faster, *i.e.*, the magnet gains speed and hence kinetic energy without expending an equivalent amount of energy. This sets up a perpetual motion machine, violating the law of conservation of energy. Thus *Lenz's law is valid and is a consequence of the law of conservation of energy.*

Examples Based on (*i*) Magnetic flux (*ii*) Laws of Electromagnetic Induction

Formulae Used

1. Magnetic flux, $\phi = BA\cos\theta = \vec{B}\cdot\vec{A}$
2. Induced emf, $\mathcal{E} = -N\dfrac{d\phi}{dt}$
3. Average induced emf, $\mathcal{E} = -N\dfrac{\phi_2 - \phi_1}{t}$
4. Induced current, $I = \dfrac{|\mathcal{E}|}{R}$.

Units Used

Magnetic field B is in tesla, flux ϕ in Wb, area A in m^2, induced emf $\mathcal{E}$ in volt., induced current I in ampere.

Example 1. *A rectangular loop of area 20 cm × 30 cm is placed in a magnetic field of 0.3 T with its plane (i) normal to the field (ii) inclined 30° to the field and (iii) parallel to the field. Find the flux linked with the coil in each case.*

Solution. $A = 20\text{ cm} \times 30\text{ cm} = 6 \times 10^{-2}\,m^2$,

$$B = 0.3\text{ T}$$

Let θ be the angle made by the field B with the normal to the plane of the coil.

(*i*) Here $\theta = 90° - 90° = 0°$

$\therefore$ $\phi = BA\cos\theta = 0.3 \times 6 \times 10^{-2} \times \cos 0°$
$= \mathbf{1.8 \times 10^{-2}\,Wb.}$

(*ii*) Here $\theta = 90° - 30° = 60°$

$\therefore$ $\phi = 0.3 \times 6 \times 10^{-2} \times \cos 60°$
$= \mathbf{0.9 \times 10^{-2}\,Wb.}$

(*iii*) Here $\theta = 90°$

$\therefore$ $\phi = 0.3 \times 6 \times 10^{-2} \times \cos 90°$
$=$ **zero.**

Example 2. *A small piece of metal wire is dragged across the gap between the pole pieces of a magnet in 0.5 s. The magnetic flux between the pole pieces is known to be 8×10^{-4} Wb. Estimate the emf induced in the wire.* [NCERT]

Solution. Here $dt = 0.5$ s,

$$d\phi = 8 \times 10^{-4} - 0 = 8 \times 10^{-4} \text{ Wb}$$

The emf induced in the wire,

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{8 \times 10^{-4}}{0.5} = \mathbf{1.6 \times 10^{-3} \ V}.$$

Example 3. *The magnetic flux through a coil perpendicular to the plane is varying according to the relation :*

$$\phi = (5t^3 + 4t^2 + 2t - 5) \ Wb$$

Calculate the induced current through the coil at $t = 2$ s, if the resistance of the coil is 5 Ω. **[Punjab 98C]**

Solution. The magnitude of induced emf set up at any instant t will be

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^3 + 4t^2 + 2t - 5) = 15t^2 + 8t + 2$$

At $t = 2$ s,

$$|\varepsilon| = 15(2)^2 + 8(2) + 2 = 60 + 16 + 2 = 78 \text{ V}$$

Resistance of the coil, $R = 5 \Omega$

Induced current,

$$I = \frac{|\varepsilon|}{R} = \frac{78}{5} = \mathbf{15.6 \ A}$$

Example 4. *A square loop of side 10 cm and resistance of 0.70 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time-interval.* [NCERT]

Solution. Clearly, the angle made by the normal to the plane (east-west plane) of the coil with the magnetic field in north-east direction is

$$\theta = 45°$$

Maximum flux through the coil,

$$\phi_{max} = BA \cos \theta = 0.1 \times (0.10 \times 0.10) \cos 45°$$

$$= \frac{10^{-3}}{\sqrt{2}} \simeq 0.7 \times 10^{-3} \text{ Wb}$$

This flux is set up in 0.7 s. So the magnitude of induced emf,

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{\phi_{max} - 0}{dt} = \frac{0.7 \times 10^{-3} \text{ Wb}}{0.7 \text{ s}}$$

$$= 10^{-3} \text{ V} = \mathbf{1 \ mV}.$$

The magnitude of induced current,

$$I = \frac{|\varepsilon|}{R} = \frac{1 \text{ mV}}{0.7 \Omega} = \mathbf{1.4 \ mA}.$$

Example 5. *A 10 Ω resistance coil has 1000 turns and at a time 5.5×10^{-4} Wb of flux passes through it. If the flux falls to 0.5×10^{-4} Wb in 0.1 second, find the emf generated in volts and the charge flown through the coil in coulombs.*

Solution. $R = 10 \Omega$, $N = 1000$, $\phi_1 = 5.5 \times 10^{-4}$ Wb, $\phi_2 = 0.5 \times 10^{-4}$ Wb, $t = 0.1$ s

Induced emf,

$$\varepsilon = -N \frac{\phi_2 - \phi_1}{t}$$

$$= -1000 \times \frac{0.5 \times 10^{-4} - 5.5 \times 10^{-4}}{0.1} = \mathbf{5 \ V}$$

Current through 10 Ω resistance coil is

$$I = \frac{\varepsilon}{R} = \frac{5}{10} = 0.5 \text{ A}$$

∴ The charge passing through the coil in 0.1 s is

$$q = It = 0.5 \times 0.1 = \mathbf{0.05 \ C}$$

Example 6. *A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T (tesla). If the induced emf is 11 V when the magnetic field is changed to 1000 T in 4 s, what is the number of turns in the coil ?* [CBSE F 94 C]

Solution. Radius of coil, $r = \frac{0.02}{2} = 0.01$ m

$B_1 = 6000$ T, $B_2 = 1000$ T, $t = 4$ s, $\varepsilon = 11$ V

Now $$|\varepsilon| = N \frac{\phi_2 - \phi_1}{t} = NA \frac{B_2 - B_1}{t}$$

$$= N \cdot \pi r^2 \cdot \frac{B_2 - B_1}{t}$$

$$\therefore \quad 11 = N \cdot \frac{22}{7} \times (0.01)^2 \frac{6000 - 1000}{4}$$

Number of turns,

$$N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = \mathbf{28}.$$

Example 7. *A wire 88 cm long bent into a circular loop is placed perpendicular to the magnetic field of flux density 2.5 Wbm^{-2}. Within 0.5 s, the loop is changed into 22 cm square and flux density is increased to 3.0 Wbm^{-2}. Calculate the value of the emf induced.*

Solution. For circular loop, $2\pi r = 88$ cm

or $$r = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm} = 0.14 \text{ m}$$

∴ Initial area of loop,

$$A_1 = \pi r^2 = \frac{22}{7} \times (0.14)^2 = 0.0616 \text{ m}^2$$

Final area of loop,

$$A_2 = (0.22)^2 = 0.0484 \text{ m}^2$$

$$B_1 = 2.5 \text{ Wbm}^{-2}, B_2 = 3.0 \text{ Wbm}^{-2}, t = 0.5 \text{ s}$$

Induced emf,

$$\varepsilon = -\left(\frac{\phi_2 - \phi_1}{t}\right) = -\left(\frac{B_2A_2 - B_1A_1}{t}\right)$$

$$= -\left(\frac{3 \times 0.0484 - 2.5 \times 0.0616}{0.5}\right)$$

$$= -\left(\frac{0.1452 - 0.154}{0.5}\right) = \frac{0.0088}{0.5} = \mathbf{0.0176\ V.}$$

Example 8. *A coil of mean area 500 cm² and having 1000 turns is held perpendicular to a uniform field of 0.4 gauss. The coil is turned through 180° in 1/10 second. Calculate the average induced emf.*

Solution. Here $A = 500\text{ cm}^2 = 500 \times 10^{-4}\text{m}^2$, $N = 1000$, $B = 0.4\text{ G} = 0.4 \times 10^{-4}\text{T}$, $t = 1/10$ s

When the coil is held perpendicular to the field, the normal to the plane of the coil makes an angle of 0° with the field B.

∴ Initial flux, $\phi_1 = BA\cos 0° = BA$

Final flux, $\phi_2 = BA\cos 180° = -BA$

Average induced emf,

$$\varepsilon = -N\left(\frac{\phi_2 - \phi_1}{t}\right) = -N\left(\frac{-BA - BA}{t}\right)$$

$$= \frac{2NBA}{t} = \frac{2 \times 1000 \times 0.4 \times 10^{-4} \times 500 \times 10^{-4}}{1/10}\text{ V}$$

$$= \mathbf{0.04\ V.}$$

Example 9. *A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is 3.0×10^{-5} T.* [NCERT ; CBSE F 15]

Solution. Here $A = \pi(0.10)^2 = \pi \times 10^{-2}\text{m}^2$, $R = 2\ \Omega$, $N = 500$, $t = 0.25$ s, $B = 3.0 \times 10^{-5}$ T

Initial flux through each turn of the coil,

$$\phi_1 = BA\cos\theta_1 = 3.0 \times 10^{-5} \times (\pi \times 10^{-2})\cos 0°$$
$$= 3\pi \times 10^{-7}\text{ Wb}$$

Final flux through each turn of the coil,

$$\phi_2 = BA\cos\theta_2 = 3.0 \times 10^{-5} \times (\pi \times 10^{-2})\cos 180°$$
$$= -3\pi \times 10^{-7}\text{ Wb}$$

Estimated value of emf in the coil,

$$\varepsilon = -N\frac{\phi_2 - \phi_1}{t}$$

$$= -500 \times \frac{-3\pi \times 10^{-7} - 3\pi \times 10^{-7}}{0.25}$$

$$= \frac{500 \times 6\pi \times 10^{-7}}{0.25} = \mathbf{3.8 \times 10^{-3}\ V}$$

Current induced in the coil,

$$I = \frac{\varepsilon}{R} = \frac{3.8 \times 10^{-3}\text{ V}}{2\ \Omega}$$

$$= \mathbf{1.9 \times 10^{-3}\ A.}$$

Example 10. *A coil of cross-sectional area A lies in a uniform magnetic field B with its plane perpendicular to the field. In this position the normal to the coil makes an angle of 0° with the field. The coil rotates at a uniform rate to complete one rotation in time T. Find the average induced emf in the coil during the interval when the coil rotates :*

(*i*) *from 0° to 90° position*

(*ii*) *from 90° to 180° position*

(*iii*) *from 180° to 270° and*

(*iv*) *from 270° to 360°.*

Solution. (*i*) *For rotation from 0° to 90°*

$$\phi_1 = BA\cos 0° = BA, \quad \phi_2 = BA\cos 90° = 0, \quad t = T/4$$

∴ Average induced emf,

$$\varepsilon = -\frac{\phi_2 - \phi_1}{t} = -\frac{0 - BA}{T/4} = \frac{\mathbf{4\,BA}}{\mathbf{T}}.$$

(*ii*) *For rotation from 90° to 180°*

$$\phi_1 = BA\cos 90° = 0, \quad \phi_2 = BA\cos 180° = -BA, \quad t = T/4$$

$$\therefore \quad \varepsilon = -\frac{-BA - 0}{T/4} = \frac{\mathbf{4\,BA}}{\mathbf{T}}.$$

(*iii*) *For rotation from 180° to 270°*

$$\phi_1 = BA\cos 180° = -BA, \quad \phi_2 = BA\cos 270° = 0, \quad t = T/4$$

$$\therefore \quad \varepsilon = -\frac{0 + BA}{T/4} = -\frac{\mathbf{4BA}}{\mathbf{T}}.$$

(*iv*) *For rotation from 270° to 360°.*

$$\phi_1 = BA\cos 270° = 0, \quad \phi_2 = BA\cos 360° = BA, \quad t = T/4$$

$$\therefore \quad \varepsilon = -\frac{BA - 0}{T/4} = -\frac{\mathbf{4\,BA}}{\mathbf{T}}.$$

As the sense of the induced emf in the second half rotation is opposite to that in the first half rotation, the induced current will change its direction after first half rotation.

Example 11. *A conducting circular loop is placed in a uniform transverse magnetic field of 0.02 T. Somehow, the radius of the loop begins to decrease at a constant rate of 1.0 mm/s. Find the emf induced in the loop at the instant when the radius is 2 cm.*

Solution. Suppose r is the radius of the loop at time t. Then magnetic flux linked with the loop is

$$\phi = \pi r^2 B$$

$$\therefore \quad |\varepsilon| = \frac{d\phi}{dt} = 2\pi r B\frac{dr}{dt}$$

Here $\frac{dr}{dt} = 1.0 \text{ mms} = 1.0 \times 10^{-3} \text{ ms}^{-1}$

When $r = 2.0 \text{ cm} = 2.0 \times 10^{-2}$ m, the magnitude of induced emf is

$$|\varepsilon| = 2 \times 3.14 \times 2.0 \times 10^{-2} \times 0.02 \times 1.0 \times 10^{-3}$$

$$= 2.5 \times 10^{-6} \text{ V} = \mathbf{2.5\ \mu V}$$

Problems For Practice

1. Find the magnetic flux linked with a rectangular coil of size $6 \text{ cm} \times 8 \text{ cm}$ placed at right angle to a magnetic field of 0.5 Wbm^{-2}. (Ans. 2.4×10^{-3} Wb)
2. A square coil of 600 turns, each side 20 cm, is placed with its plane inclined at 30° to a uniform magnetic field of $4.5 \times 10^{-4} \text{Wbm}^{-2}$. Find the flux through the coil. (Ans. 5.4×10^{-3} Wb)
3. The magnetic flux threading a coil changes from 12×10^{-3} Wb to 6×10^{-3} Wb in 0.01 s. Calculate the induced emf. (Ans. 0.6 V)
4. A magnetic field of flux density 1.0 Wbm^{-2} acts normal to a 80 turn coil of 0.01 m^2. Find the emf induced in it, if this coil is removed from the field in 0.1 s. **[Haryana 02]** (Ans. 8V)
5. A 70 turn coil with average diameter of 0.02 m is placed perpendicular to magnetic field of 9000 T. If the magnetic field is changed to 6000 T in 3 s, what is the magnitude of the induced emf ? **[CBSE F 94C]** (Ans. 2.2 V)
6. A magnetic field of flux density 10 T acts normal to a 50 turn coil of 100 cm^2 area. Find the emf induced in it if the coil is removed from the field in 1/20 s. (Ans. 100 V)
7. A coil has 1000 turns and 500 cm^2 as its area. It is placed at right angles to a magnetic field of 2×10^{-5} Wb m^{-2}. The coil is rotated through 180° in 0.2 s. Find the average emf induced in the coil. (Ans. 10 mV)
8. A coil of area 0.04 m^2 having 1000 turns is suspended perpendicular to a magnetic field of $5.0 \times 10^{-5} \text{ Wbm}^{-2}$. It is rotated through 90° in 0.2 s. Calculate the average emf induced in it. (Ans. 0.01 V)
9. A wire 40 cm long bent into a rectangular loop $15 \text{ cm} \times 5 \text{ cm}$ is placed perpendicular to the magnetic field whose flux density is 0.8 Wbm^{-2}. Within 1.0 second, the loop is changed into a 10 cm square and flux density increases to 1.4 Wbm^{-2}. Calculate the value of induced emf. (Ans. – 0.008 V)
10. An air-cored solenoid of length 50 cm and area of cross-section 28 cm^2 has 200 turns and carries a current of 5.0 A. On switching off, the current decreases to zero within a time interval of 1.0 ms. Find the average emf induced across the ends of the open switch in the circuit. (Ans. 1.4 V)
11. A closed coil consists of 500 turns on a rectangular frame of area 4.0 cm^2 and has a resistance of $50\,\Omega$. It is kept with its plane perpendicular to a uniform magnetic field of 0.2 Wb m^{-2}. Calculate the amount of charge flowing through the coil when it is turned over (rotated through 180°). Will this answer depend on the speed with which the coil is rotated ? (Ans. 1.6×10^{-3} C, No)
12. The magnetic flux through a coil perpendicular to its plane and directed into paper is varying according to the relation $\phi = (5t^2 + 10t + 5)$ milliweber. Calculate the emf induced in the loop at $t = 5$ s. (Ans. 0.06 V)

HINTS

1. $\phi = BA \cos 0° = 0.5 \times 0.06 \times 0.08 \times 1 = \mathbf{2.4 \times 10^{-3}\ Wb}$.
2. $\phi = NBA \cos \theta$
$= 600 \times 4.5 \times 10^{-4} \times 0.20 \times 0.20 \times \cos(90° - 30°)$
$= \mathbf{5.4 \times 10^{-3}\ Wb}$.
3. $\varepsilon = -\frac{\phi_2 - \phi_1}{t} = -\frac{6 \times 10^{-3} - 12 \times 10^{-3}}{0.01} = \mathbf{0.6\ V}$.
4. $\varepsilon = -NA\frac{B_2 - B_1}{t} = -80 \times 0.01 \times \frac{0 - 1.0}{0.1} = \mathbf{8\ V}$.
5. $\varepsilon = -N \cdot \pi r^2 \frac{B_2 - B_1}{t}$
$= -70 \times \frac{22}{7} \times (0.01)^2 \cdot \frac{6000 - 9000}{3} = \mathbf{2.2\ V}$.
6. $\varepsilon = -NA\frac{B_2 - B_1}{1}$
$= 50 \times 100 \times 10^{-4} \times \frac{10 - 0}{1/20} = \mathbf{100\ V}$.
7. Proceed as in Example 8 on page 6.7.
8. $\varepsilon = -N\frac{BA\cos 90° - BA\cos 0°}{t} = -NBA \times \frac{0-1}{t}$
$= -1000 \times 5.0 \times 10^{-5} \times 0.04 \times \frac{0-1}{0.2} = \mathbf{0.01\ V}$.
9. Here $B_1 = 0.8 \text{ Wbm}^{-2}$,
$A_1 = 15 \times 5 \text{ cm}^2 = 75 \times 10^{-4} \text{m}^2$,
$B_2 = 1.4 \text{ Wbm}^{-2}$,
$A_2 = 10 \times 10 \text{ cm}^2 = 100 \times 10^{-4} \text{m}^2$, $t = 1$s
$\varepsilon = -\frac{B_2 A_2 - B_1 A_1}{t}$
$= -\frac{1.4 \times 100 \times 10^{-4} - 0.8 \times 75 \times 10^{-4}}{1} = \mathbf{-0.008\ V}$.

10. Proceed as in Exercise 6.15 on page 6.54.

11. $\Delta\phi = BA\cos 180° - BA\cos 0° = -2BA$

$$\mathcal{E} = -N\frac{d\phi}{dt} = -N\frac{\Delta\phi}{\Delta t} = \frac{2NBA}{\Delta t}$$

$$q = I\Delta t = \frac{\mathcal{E}}{R}.\Delta t = \frac{2NBA}{R}$$

$$= \frac{2\times 500\times 0.2\times 4.0\times 10^{-4}}{50}$$

$$= \mathbf{1.6\times 10^{-3}\ C.}$$

12. Here $\varphi = (5t^2 + 10t + 5)$ mWb

$$= (5t^2 + 10t + 5)\times 10^{-3}\,\text{Wb}$$

$$|\mathcal{E}| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^2 + 10t + 5)\times 10^{-3}$$

$$= (10t + 10)\times 10^{-3}\,\text{V}$$

$\therefore$ At $t = 5$ s, $|\mathcal{E}| = (10\times 5 + 10)\times 10^{-3} = \mathbf{0.06\ V.}$

6.6 MOTIONAL EMF FROM FARADAY'S LAWS

7. *What is motional emf ? Deduce an expression for the emf induced across the ends of a conductor moving in a perpendicular magnetic field.*

Motional emf from Faraday's law : Induced emf by change of area of the coil linked with the magnetic field. *The emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf.* As shown in Fig. 6.8, consider a conductor PQ of length l free to move on U-shaped conducting rails situated in a uniform and time independent magnetic field B, directed normally into the plane of paper. The conductor PQ is moved inwards with a speed v. As the conductor slides towards left, the area of the rectangular loop $PQRS$ decreases. This decreases the magnetic flux linked with the closed loop. Hence an emf is set up across the ends of conductor PQ because of which an induced current flows in the circuit along the path $PQRS$. The direction of induced current can by determined by using Fleming's right hand rule, stated in the next section.

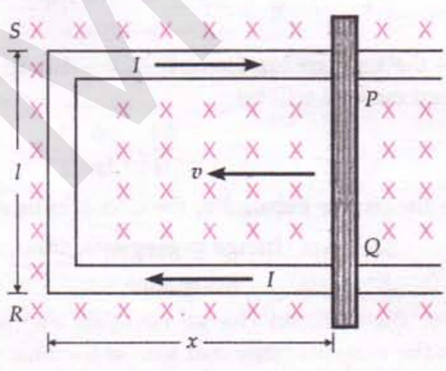


Fig. 6.8 Induced current by changing area of the rectangular loop.

Suppose a length x of the loop lies inside the magnetic field at any instant of time t. Then the magnetic flux linked with the rectangular loop $PQRS$ is

$$\phi = BA = Blx$$

According to Faraday's law of electromagnetic induction, the induced emf is

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt}$$

or $\mathcal{E} = Blv$

where $dx/dt = -v$, because the velocity v is in the decreasing direction of x. The induced emf Blv is called ***motional emf*** because this *emf is induced due to the motion of a conductor in a magnetic field.*

6.7 FLEMING'S RIGHT HAND RULE

8. *State a rule to determine the direction of current induced due to the motion of a conductor in a perpendicular magnetic field.*

Fleming's right hand rule. This rule gives the direction of induced current set up in a conductor moving perpendicular to a magnetic field and can be stated as follows :

If we stretch the thumb and the first two fingers of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor ; then the central finger points in the direction of current induced in the conductor.

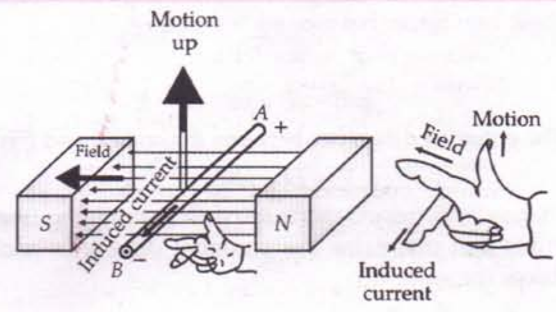


Fig. 6.9 Fleming's right hand rule.

This rule is also called the ***dynamo*** or ***generator rule*** and has wide practical applications.

6.8 MOTIONAL EMF FROM LORENTZ FORCE AND ENERGY CONSIDERATION

9. *Deduce an expression for the motional emf by considering the Lorentz force acting on the free charge carriers of a conductor moving in a perpendicular magnetic field. Also deduce expressions for the induced current, force necessary to pull the conductor, power delivered by the external source, and power dissipated as Joule loss. Hence discuss the energy conservation.*

Motional emf from Lorentz force. A conductor has a large number of free electrons. When it moves through a magnetic field, a Lorentz force acting on the free electrons can set up a current. Fig. 6.10 shows a rectangular conductor in which arm PQ is free to move. It is placed in a uniform magnetic field B, directed normally into the plane of paper. As the arm PQ is moved towards left with a speed v, the free

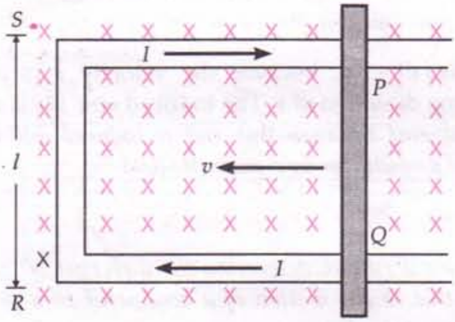


Fig. 6.10 Motional emf.

electrons of PQ also move with the same speed towards left. The electrons experience a magnetic Lorentz force, $F_m = qvB$. According to Fleming's left hand rule, this force acts in the direction QP and hence the free electrons will move towards P. A negative charge accumulates at P and a positive charge at Q. An electric field E is set up in the conductor from Q to P. This field exerts a force, $F_e = qE$ on the free electrons. The accumulation of charges at the two ends continues till these two forces balance each other, *i.e.,*

$$F_m = F_e$$

or
$$qvB = qE \quad \text{or} \quad vB = E$$

The potential difference between the ends Q and P is

$$V = El = vBl$$

Clearly, it is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf,

$$\varepsilon = Blv$$

As this emf is produced due to the motion of a conductor, so it is called a ***motional emf***.

Current induced in the loop. Let R be the resistance of the movable arm PQ of the rectangular loop $PQRS$ shown in Fig. 6.10. Suppose the total resistance of the remaining arms QR, RS and SP is negligible compared to R. Then the current in the loop will be

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}$$

Force on the movable arm. The conductor PQ of length l and carrying current I experiences force F in the perpendicular magnetic field B.

The force is given by

$$F = IlB \sin 90° = \left(\frac{Blv}{R}\right) lB = \frac{B^2 l^2 v}{R}$$

This force (due to induced current) acts in the outward direction opposite to the velocity of the arm in accordance with Lenz's law. Hence to move the arm with a constant velocity v, it should be pulled with a constant force F.

Power delivered by the external force. The power supplied by the external force to maintain the motion of the movable arm is

$$P = Fv = \frac{B^2 l^2 v^2}{R}$$

Power dissipated as Joule loss. The power dissipated in the loop as Joule heating loss is

$$P_J = I^2 R = \left(\frac{Blv}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}.$$

Clearly, $P_J = P$. Thus, the mechanical energy expended to maintain the motion of the movable arm is first converted into electrical energy (the induced emf) and then to thermal energy. This is consistent with the law of conservation of energy. This fact allows us to represent the electromagnetic set up of Fig. 6.10 by the equivalent electrical circuit shown in Fig. 6.11.

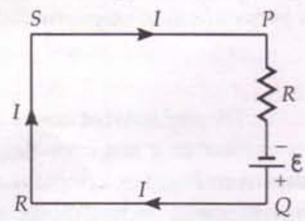


Fig. 6.11 Equivalent electrical circuit of the electromagnetic set up of Fig. 6.10.

6.9 RELATION BETWEEN INDUCED CHARGE AND CHANGE IN MAGNETIC FLUX

10. *Prove that the induced charge does not depend on the rate of change of magnetic flux.*

Relation between induced charge and change in magnetic flux. According to Faraday's law, the magnitude of induced emf, $|\varepsilon| = \frac{\Delta\phi}{\Delta t}$

If R is the total resistance of the closed loop, then the induced current will be

$$I = \frac{\varepsilon}{R} \quad \text{or} \quad \frac{\Delta q}{\Delta t} = \frac{\Delta\phi}{\Delta t} \cdot \frac{1}{R}$$

Hence the charge induced in the circuit in time Δt,

$$\Delta q = \frac{\Delta\phi}{R} = \frac{\text{Net change in magnetic flux}}{\text{Resistance}}$$

Clearly, the induced charge depends on the net change in the magnetic flux and not on the time interval Δt of the flux change. Thus the induced charge does not depend on the rate of change of magnetic flux.

Examples Based on Motional EMF

Formulae Used

1. The emf induced in a conductor of length l moving with velocity v perpendicular to field B, $\mathcal{E} = Blv$.
2. Induced emf developed between the two ends of rod rotating at its one end in perpendicular magnetic field, $\mathcal{E} = \frac{1}{2} Bl^2 \omega$

Units Used

Field B is in tesla or Wbm^{-2}, length l in metre, velocity v in ms^{-1}, angular speed ω in rad s^{-1} and emf $\mathcal{E}$ in volt.

Example 12. *An aircraft with a wing span of 40 m flies with a speed of 1080 kmh^{-1} in the eastward direction at a constant altitude in the northern hemisphere, where the vertical component of earth's magnetic field is 1.75×10^{-5} T. Find the emf that develops between the tips of the wings.* [NCERT]

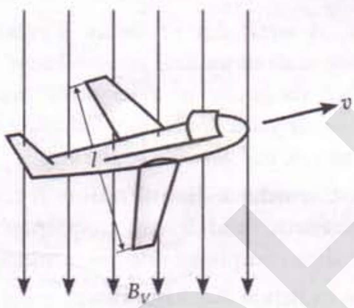


Fig. 6.12

Solution. The metallic part between the wing- tips can be treated as a single conductor cutting flux- lines due to vertical component of earth's magnetic field. So emf is induced between the tips of its wings.

Here $l = 40$ m, $B_V = 1.75 \times 10^{-5}$ T

$$v = 1080 \text{ kmh}^{-1} = \frac{1080 \times 1000}{3600} \text{ ms}^{-1}$$

$$= 300 \text{ ms}^{-1}$$

$$\therefore \quad \mathcal{E} = B_V lv = 1.75 \times 10^{-5} \times 40 \times 300 = \mathbf{0.21\ V}.$$

Example 13. *A jet plane is travelling west at 450 ms^{-1}. If the horizontal component of earth's magnetic field at that place is 4×10^{-4} tesla and the angle of dip is 30°, find the emf induced between the ends of wings having a span of 30 m.* [CBSE OD 08]

Solution. Here $l = 30$ m, $v = 450\ ms^{-1}$, $\delta = 30°$, $B_H = 4 \times 10^{-4}$ T

The wings of the airplane cut the flux-lines of the vertical component of earth's magnetic field, which is given by

$$B_V = B_H \tan\delta = 4 \times 10^{-4} \tan 30° = 4 \times 10^{-4} \times 0.577 \text{ T}$$

The emf induced across the tips of the wings is

$$\mathcal{E} = B_V lv = 4 \times 10^{-4} \times 0.577 \times 30 \times 450 = \mathbf{3.12\ V.}$$

Example 14. *A railway track running north-south has two parallel rails 1.0 m apart. Calculate the value of induced emf between the rails, when a train passes at a speed of 90 kmh^{-1}. The horizontal component of earth's magnetic field at that place is 0.3×10^{-4} Wbm^{-2} and angle of dip is 60°.* [Haryana 01]

Solution. Here $l = 1.0$ m, $B_H = 0.3 \times 10^{-4} \text{Wb m}^{-2}$, $\delta = 60°$

$$B_V = B_H \tan\delta = 0.3 \times 10^{-4} \tan 60°$$

$$= 0.3 \times 10^{-4} \times 1.732 = 0.52 \times 10^{-4} \text{ Wbm}^{-2}.$$

$$v = 90 \text{ kmh}^{-1} = \frac{90 \times 1000}{3600} = 25 \text{ ms}^{-1}.$$

$$\therefore \quad \mathcal{E} = B_V lv = 0.52 \times 10^{-4} \times 1.0 \times 25 = \mathbf{1.3 \times 10^{-3}\ V.}$$

Example 15. *A conductor of length 1.0 m falls freely under gravity from a height of 10 m so that it cuts the lines of force of the horizontal component of earth's magnetic field of 3×10^{-5} Wbm^{-2}. Find the emf induced in the conductor.*

Solution. The velocity v attained by the conductor as it falls through a height of 10 m is given by

$$v^2 = u^2 + 2gs = 0 + 2 \times 9.8 \times 10 = 4 \times 49$$

$$\therefore \quad v = 2 \times 7 = 14 \text{ ms}^{-1}$$

Induced emf,

$$\mathcal{E} = B_H lv = 3 \times 10^{-5} \times 1.0 \times 14 = \mathbf{4.2 \times 10^{-4}\ V.}$$

Example 16. *Twelve wires of equal lengths (each 10 cm) are connected in the form of a skeleton-cube. (i) If the cube is moving with a velocity of 5 ms^{-1} in the direction of a magnetic field of 0.05 Wbm^{-2}, find the emf induced in each arm of the cube. (ii) If the cube moves perpendicular to the field, what will be the induced emf in each arm ?*

Solution. For the generation of motional emf : B, l and v must be in mutually perpendicular directions. If any two of these quantities are parallel, emf is not induced.

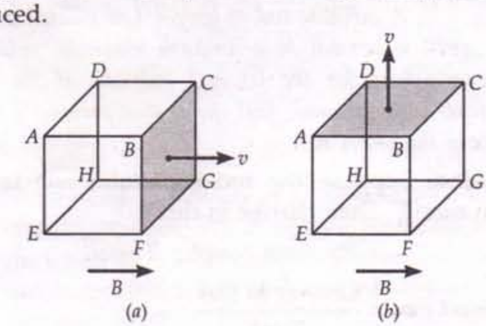


Fig. 6.13

(*i*) In Fig. 6.13(*a*), the velocity of any conductor is parallel to the field *B*, so no emf is induced in any conductor.

(*ii*) In Fig. 6.13(*b*), the arms *AE*, *BF*, *CG* and *DH* are parallel to the velocity *v*, no emf is induced in these arms. Also, the arms *AB*, *DC*, *EF* and *HG* are parallel to the field *B*, so no emf is induced in these arms.

The arms *AD*, *BC*, *EH* and *FG* are perpendicular to both *B* and *v*. Hence emf is induced in each of these arms and is given by

$$\varepsilon = Blv = 0.05 \times 10 \times 10^{-2} \times 5 = \mathbf{2.5 \times 10^{-2}\ V.}$$

Example 17. *Fig. 6.14 shows a conducting rod PQ in contact with metal rails RP and SQ which are 25 cm apart in a uniform magnetic field of flux density 0.4 T acting perpendicular to the plane of the paper. Ends R and S are connected through a 5 Ω resistance. What is the emf when the rod moves to the right with a velocity of 5 ms^{-1}? What is the magnitude and direction of the current through 5 Ω resistance? If the rod moves to the left with the same speed, what will be the new current and its direction?* [ISCE 95]

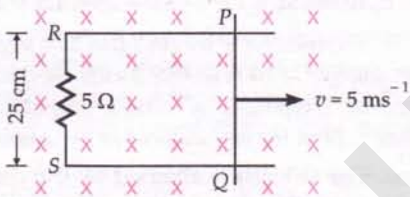


Fig. 6.14

Solution. Here $B = 0.4$ T, $v = 5$ ms^{-1}, $l = 25$ cm $= 0.25$ m

Induced emf, $\varepsilon = Blv = 0.4 \times 0.25 \times 5 = \mathbf{0.5\ V}$

Current, $I = \dfrac{\varepsilon}{R} = \dfrac{0.5\ \text{V}}{5\ \Omega} = \mathbf{0.1\ A}$

Applying Fleming right hand rule, the induced current flows from *Q* to *P*, *i.e., from the end R to S* through the 5 Ω resistance.

If the rod moves to the left with the same speed, then the current of 0.1A will flow through 5 Ω resistance ***from the end S to R.***

Example 18. *A metallic rod of length L is rotated at an angular speed ω normal to a uniform magnetic field B. Derive expressions for the (i) emf induced in the rod (ii) current induced and (iii) heat dissipation, if the resistance of the rod is R.* [CBSE D 08]

Solution. Suppose the rod completes one revolution in time *T*. Then change in flux

$$= B \times \text{Area swept} = B \times \pi L^2$$

$$\text{Induced emf} = \frac{\text{Change in flux}}{\text{Time}}$$

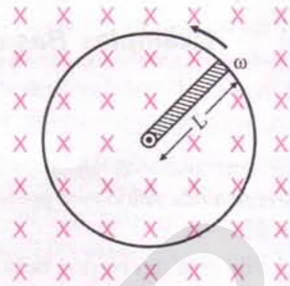


Fig. 6.15

or $\quad \varepsilon = \dfrac{B \times \pi L^2}{T} = B\pi L^2 f \qquad \left[\because T = \dfrac{1}{f}\right]$

As $f = \dfrac{\omega}{2\pi}$, therefore

$$\varepsilon = B\pi L^2 . \frac{\omega}{2\pi} = \frac{1}{2} BL^2\omega$$

Induced current,

$$I = \frac{\varepsilon}{R} = \frac{1}{2}\frac{BL^2\omega}{R}$$

Heat dissipation in time *t*,

$$Q = \frac{\varepsilon^2 t}{R} = \frac{\mathbf{1}}{\mathbf{4}}\frac{\mathbf{B^2 L^4 \omega^2 t}}{\mathbf{R}}.$$

Example 19. *A metal disc of radius R rotates with an angular velocity ω about an axis perpendicular to its plane passing through its centre in a magnetic field B acting perpendicular to the plane of the disc. Calculate the induced emf between the rim and the axis of the disc.*

Solution. Consider a disc of radius *R* rotating in a transverse magnetic field *B* with frequency *f*. In time period *T*, the disc completes one revolution.

$\therefore$ Change in flux = $B \times$ Area swept = $B \times \pi R^2$

$$\text{Induced emf} = \frac{\text{change in flux}}{\text{Time}}$$

$$\varepsilon = \frac{B\pi R^2}{T} = B\pi R^2 f \qquad \left[\because \frac{1}{f} = T\right]$$

As $f = \dfrac{\omega}{2\pi}$, $\quad \therefore \varepsilon = B \times \pi R^2 . \dfrac{\omega}{2\pi} = \dfrac{\mathbf{1}}{\mathbf{2}} \mathbf{BR^2\omega}$

Example 20. *A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev / min in a plane normal to the horizontal component of earth's magnetic field B_H at a place. If $B_H = 0.4$ G at the place, what is the induced emf between the axle and the rim of the wheel? Note that 1 G = 10^{-4} T.* [NCERT]

Solution. Here $L = 0.50$ m, $B = 0.40\,\text{G} = 0.40 \times 10^{-4}$ T

$$f = 120\ \frac{\text{rev}}{\text{min}} = \frac{120}{60}\frac{\text{rev}}{\text{sec}} = 2\ \text{rps}$$

Induced emf,

$$\varepsilon = B\pi L^2 f = 0.40 \times 10^{-4} \times 3.14 \times (0.50)^2 \times 2$$

$$= \mathbf{6.28 \times 10^{-5}\,V.}$$

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

Example 21. *When a wheel with metal spokes 1.0 m long rotates in a magnetic field of flux density 2×10^{-4} T normal to the plane of the wheel, an emf of $\pi \times 10^{-2}$ V is induced between the rim and the axle of the wheel. Find the rate of revolution of the wheel.*

Solution. Here $L = 1.0$ m, $B = 2 \times 10^{-4}$ T, $\varepsilon = \pi \times 10^{-2}$ V, $f = ?$

As $\quad \varepsilon = B \pi L^2 f$

$$\therefore \quad f = \frac{\varepsilon}{B \pi L^2} = \frac{\pi \times 10^{-2}}{2 \times 10^{-4} \times \pi \times (1.0)^2} = 50 \textbf{ rps.}$$

Example 22. *A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 1 m, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field of 1 T and parallel to the axis is present everywhere. What is the emf between the centre and the metallic ring ? [Fig. 6.15]* [NCERT]

Solution. Here $L = 1$ m, $f = 50$ rps, $B = 1.0$ Wbm^{-2}

$$\therefore \quad \varepsilon = B \pi L^2 f$$
$$= 1.0 \times 3.14 \times (1)^2 \times 50 = \textbf{157 V.}$$

Example 23. *A circular copper disc 10 cm in radius rotates at 20π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2 T acts perpendicular to the disc. (i) Calculate the potential difference developed between the axis of the disc and the rim. (ii) What is the induced current, if the resistance of the disc is 2 ohm ?* [CBSE OD 01]

Solution. Here $R = 10$ cm $= 0.10$ m, $\omega = 20\pi$ rad s^{-1}, $B = 0.2$ T

(*i*) P.D. developed between the axis and the rim

$$\varepsilon = \frac{1}{2} BR^2 \omega = \frac{1}{2} \times 0.2 \times (0.10)^2\, 20\pi$$
$$= \textbf{0.0628 V}$$

(*ii*) Induced current,

$$I = \frac{\varepsilon}{R} = \frac{0.0628}{2} = \textbf{0.0314 A.}$$

Example 24. *A 0.5 m long metal rod PQ completes the circuit as shown in Fig. 6.16. The area of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the total circuit is 3 Ω, calculate the force needed to move the rod in the direction as indicated with a constant speed of 2 ms^{-1}.* [CBSE OD 06]

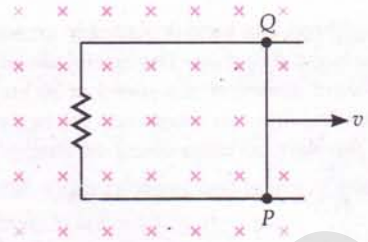


Fig. 6.16

Solution. Here $\varepsilon = Blv$ and $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$

$$F = IlB \sin 90° = \frac{Blv}{R} \cdot lB = \frac{B^2 l^2 v}{R}$$

But $l = 0.5$ m, $B = 0.15$ T, $R = 3\,\Omega$, $v = 2$ ms^{-1}

$$\therefore \quad F = \frac{(0.15)^2 \times (0.5)^2 \times 2}{3} = \textbf{0.00375 N.}$$

Problems For Practice

1. A straight conductor 1 metre long moves at right angles to both, its length and a uniform magnetic field. If the speed of the conductor is 2.0 ms^{-1} and the strength of the magnetic field is 10^4 gauss, find the value of induced emf in volt. **[Punjab 96]** (Ans. 2 V)
2. If a 10 m long metallic bar moves in a direction at right angle to a magnetic field with a speed of 5.0 ms^{-1}, 25 V emf is induced in it. Find the value of the magnetic field intensity. **[Punjab 99]** (Ans. 0.5 T)
3. A horizontal telephone wire 1 km long is lying east-west in earth's magnetic field. It falls freely to the ground from a height of 10 m. Calculate the emf induced in the wire on striking the ground. Given $B_H = 0.32$ G. (Ans. 0.448 V)
4. A horizontal wire 24 cm long falls in the field of flux density 0.8 T. Calculate the emf induced in it at the end of 3 s, after it was dropped from rest. Suppose the wire moves perpendicular to its length as well as to magnetic field. Take $g = 9.8$ ms^{-2}. (Ans. 5.6 V)
5. The two rails of a railway track insulated from each other and the ground are connected to a millivoltmeter. What is the reading of the voltmeter when a train travels at a speed of 180 kmh^{-1} along the track, given that the vertical component of the earth's magnetic field is 0.2×10^{-4} Wb m^{-2} and the rails are separated by 1 m ? **[IIT]** (Ans. 1 mV)
6. A jet plane is moving at a speed of 1000 km h^{-1}. What is the potential difference across the ends of its wings 20 m long. Given total intensity of earth's magnetic field is 3.5×10^{-4} tesla and angle of dip at the place is 30°. (Ans. 0.97 volt)

7. A straight rod 2 m long is placed in an aeroplane in the east-west direction. The aeroplane lifts itself in the upward direction at a speed of 36 km h^{-1}. Find the potential difference between the two ends of the rod if the vertical component of earth's magnetic field is $\frac{1}{4\sqrt{3}}$ gauss and angle of dip = 30°.

(Ans. 5×10^{-4} V)

8. Figure shows a rectangular conducting loop *PQRS* in which the arm *PQ* is free to move. A uniform magnetic field acts in the direction perpendicular to the plane of the loop. Arm *PQ* is moved with a velocity *v* towards the arm *RS*. Assuming that the arms *QR*, *RS* and *SP* have negligible resistances and the moving arm *PQ* has the resistance *R*, obtain the expression for (*i*) the current in the loop (*ii*) the force and (*iii*) the power required to move the arm *PQ*. **[CBSE D 13]**

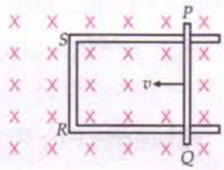


Fig. 6.17

9. A rectangular loop *PQMN* with movable arm *PQ* of length 10 cm and resistance 2 Ω is placed in a uniform magnetic field of 0.1 T acting perpendicular to the plane of the loop as is shown in the figure. The

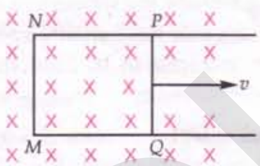


Fig. 6.18

resistances of the arms *MN*, *NP* and *MQ* are negligible. Calculate the (*i*) emf induced in the arm *PQ* and (*ii*) current induced in the loop when arm *PQ* is moved with velocity 20 ms^{-1}. **[CBSE D 14C]**

(Ans. 0.2 V, 0.1 A)

10. A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the place is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased ? **[CBSE OD 13]**

(Ans. 3.14×10^{-5} V)

11. A fan blade of length 2*a* rotates with frequency *f* cycles per second perpendicular to a magnetic field *B*. Find the p.d. between the centre and the end of the blade. (Ans. $-\pi Ba^2 f$)

12. In a ceiling fan, each blade rotates in a circle of radius 0.5 m. If the fan makes 20 revolutions per second and if the vertical component of earth's field is 8×10^{-5} Wb m^{-2}, calculate the p.d. developed between the ends of each blade. (Ans. 0.001 T)

13. A metal disc of radius 200 cm is rotated at a constant angular speed of 60 rad s^{-1} in a plane at right angles to an external field of magnetic induction 0.05 Wbm^{-2}. Find the emf induced between the centre and a point on the rim. **[Punjab 91]**

(Ans. 6 V)

14. A copper disc of radius 10 cm placed with its plane normal to a uniform magnetic field completes 1200 rotations per minute. If induced emf between the centre and the edge of the disc is 6.284 mV, find the intensity of the magnetic field. Take π = 3.142

(Ans. 10^{-2} T)

HINTS

1. Here $l = 1$ m, $v = 2.0$ ms^{-1}, $B = 10^4$ G = 1 T

$\therefore \quad \mathcal{E} = Blv = 1 \times 1 \times 2 = \mathbf{2V}.$

2. $B = \frac{\mathcal{E}}{lv} = \frac{25}{10 \times 5} = \mathbf{0.5\ T}.$

3. Here $v^2 = u^2 + 2gs = 0 + 2 \times 9.8 \times 10 = 4 \times 49$

or $v = 2 \times 7 = 14\ ms^{-1}$

$\mathcal{E} = B_H\, lv = 0.32 \times 10^{-4} \times 1 \times 10^3 \times 14$

$= \mathbf{0.448\ V}.$

4. Here $v = u + at = 0 + 9.8 \times 3 = 29.4\ ms^{-1}$

$\therefore \quad \mathcal{E} = Blv = 0.8 \times 0.24 \times 29.4 = \mathbf{5.6\ V}.$

5. Here $B_V = 0.2 \times 10^{-4}\ Wbm^{-2}$, $l = 1$ m

$$v = 180\ kmh^{-1} = \frac{180 \times 1000}{3600}\ ms^{-1} = 50\ ms^{-1}$$

∴ Induced emf,

$\mathcal{E} = B_V lv = 0.2 \times 10^{-4} \times 1 \times 50$

$= 10^{-3}\ V = \mathbf{1\ mV}.$

Thus the reading of the milli-voltmeter is 1 mV.

6. Proceed as in Exercise 6.10 on page 6.52.

7. Here $l = 2$ m, $v = 36\ kmh^{-1} = 10\ ms^{-1}$, $\delta = 30°$

$$B_V = \frac{1}{4\sqrt{3}}\ G = \frac{10^{-4}}{4\sqrt{3}}\ T$$

$$B_H = \frac{B_V}{\tan \delta} = \frac{10^{-4}}{4\sqrt{3} \tan 30°} = \frac{10^{-4}}{4\sqrt{3} \cdot \frac{1}{\sqrt{3}}} = \frac{1}{4} \times 10^{-4}\ T$$

$$\therefore \quad \mathcal{E} = B_H lv = \frac{1}{4} \times 10^{-4} \times 2 \times 10 = \mathbf{5 \times 10^{-4}\ V}.$$

8. (*i*) $I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$

(*ii*) $F = IlB \sin 90° = \left(\frac{Blv}{R}\right) lB = \frac{B^2 l^2 v}{R}$

(*iii*) $P = Fv = \frac{B^2 l^2 v^2}{R}$.

9. Here

$B = 0.1$ T, $l = 10$ cm $= 0.10$ m, $v = 20\,\text{ms}^{-1}$, $R = 2\,\Omega$

(*i*) $\mathcal{E} = Blv = 0.1 \times 0.10 \times 20 = \mathbf{0.2\ V}$

(*ii*) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.2}{2} = \mathbf{0.1\ A}$

10. Here $l = 50$ cm $= 0.50$ m,

$$f = 120\frac{\text{rev}}{\text{min}} = \frac{120\ \text{rev}}{60\ \text{sec}} = 2\,\text{rps}$$

$$B_H = B\cos\delta = 0.4\cos 60° = 0.2\text{G} = 0.2 \times 10^{-4}\,\text{T}$$

Induced emf,

$$\mathcal{E} = B_H \pi l^2 f = 0.2 \times 10^{-4} \times 3.14 \times (0.50)^2 \times 2 = \mathbf{3.14 \times 10^{-5}\ V.}$$

11. $\mathcal{E} = -\dfrac{d\phi}{dt} = -\dfrac{d}{dt}(BA) = -B\dfrac{dA}{dt}$

In time period T, the area swept by the blades is πa^2.

$$\therefore\ \mathcal{E} = -B.\frac{\pi a^2}{T} = -B.\frac{\pi a^2}{1/f} = -\pi \boldsymbol{Ba^2 f}$$

12. Use $\mathcal{E} = B \times \pi r^2 \times f$.

13. $\mathcal{E} = \dfrac{1}{2}Br^2\omega = \dfrac{1}{2} \times 0.05 \times 2^2 \times 60 = \mathbf{6\ V.}$

14. Here $r = 10$ cm $= 0.10$ m, $f = 1200/60 = 20\,\text{s}^{-1}$,

$$\mathcal{E} = 6.284 \times 10^{-3}\,\text{V} = B \times \pi r^2 \times f$$

$$B = \frac{\mathcal{E}}{\pi r^2 f} = \frac{6.284 \times 10^{-3}}{3.142 \times (0.10)^2 \times 20} = \mathbf{10^{-2}\ T.}$$

6.10 METHODS OF GENERATING INDUCED EMF

11. *Discuss the various methods of generating induced emf.*

Methods of generating induced emf. An induced emf can be produced by changing the magnetic flux linked with a circuit. The magnetic flux,

$$\phi = BA\cos\theta$$

can be changed by one of the following methods :

1. Changing the magnetic field B,
2. Changing the area A of the coil, and
3. Changing the relative orientation θ of B and A.

1. Induced emf by changing the magnetic field *B*. We have already learnt in section 6.3 how an induced emf is set up in a coil on changing the magnetic flux through it by (*i*) moving a magnet towards a stationary coil, (*ii*) moving a coil towards a stationary magnet and (*iii*) varying current in the neighbouring coil.

2. Induced emf by changing the area of the coil. Consider a conductor CD of length l moving with a velocity v towards right on U-shaped conducting rails situated in a magnetic field B, as shown in Fig 6.19. The field is uniform and points into the plane of the paper. As the conductor slides, the area of the circuit changes from $ABCD$ to $ABC'D'$ in time dt.

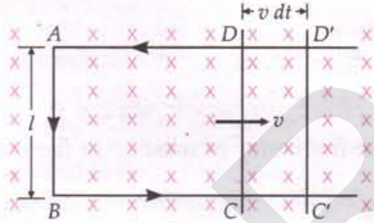


Fig. 6.19 Induced emf by changing area of the loop.

The increase in flux,

$$d\phi = B \times \text{change in area} = B \times \text{area } CDD'C' = B\,.\,l\,.\,vdt$$

This sets up induced emf in the loop of magnitude,

$$|\mathcal{E}| = \frac{d\phi}{dt} = Blv$$

According to Fleming's right hand rule, the induced current flows in the anticlockwise direction.

3. Induced emf by changing relative orientation of the coil and the magnetic field : Theory of *AC* generator. Consider a coil $PQRS$ free to rotate in a uniform magnetic field $\vec{B}$. The axis of rotation of the coil is perpendicular to the field $\vec{B}$. The flux through the coil, when its normal makes an angle θ with the field, is given by

$\phi = BA\cos\theta$, where A is the face area of the coil.

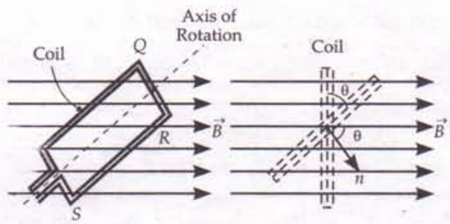


Fig. 6.20 (*a*) Rotating coil in a magnetic field.

If the coil rotates with an angular velocity ω and turns through an angle θ in time t, then

$$\theta = \omega t \qquad \therefore \qquad \phi = BA\cos\omega t$$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t) = BA\omega\sin\omega t$$

If the coil has N turns, then the total induced emf will be

$$\mathcal{E} = NBA\,\omega\sin\omega t$$

Thus the induced emf varies sinusoidally with time t. The value of induced emf is maximum when $\sin \omega t = 1$ or $\omega t = 90°$, *i.e.*, when the plane of the coil is *parallel* to the field $\vec{B}$. Denoting this maximum value by $\mathcal{E}_0$, we have

$$\mathcal{E}_0 = NBA\omega$$

$$\therefore \quad \mathcal{E} = \mathcal{E}_0 \sin \omega t = \mathcal{E}_0 \sin 2\pi ft$$

where f is the frequency of rotation of the coil.

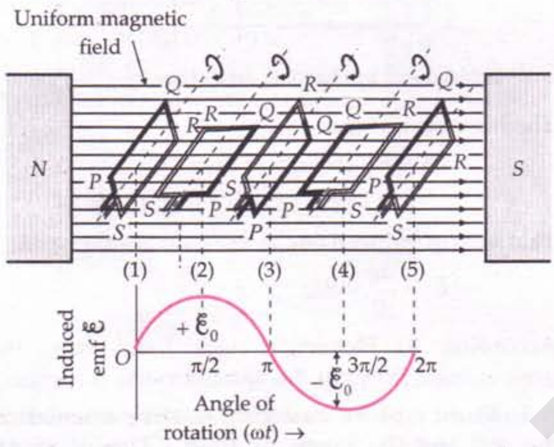


Fig. 6.20 (*b*) Induced emf in a rotating coil.

Figure 6.20(*b*) shows how the induced emf $\mathcal{E}$ between the two terminals of the coil varies with time. We consider the following special cases :

1. When $\omega t = 0°$, the plane of the coil is perpendicular to $\vec{B}$,

$$\sin \omega t = \sin 0° = 0, \quad \text{so that } \mathcal{E} = 0$$

2. When $\omega t = \frac{\pi}{2}$, the plane of the coil is parallel to field $\vec{B}$,

$$\sin \omega t = \sin \frac{\pi}{2} = 1, \quad \text{so that } \mathcal{E} = \mathcal{E}_0$$

3. When $\omega t = \pi$, the plane of the coil is again perpendicular to B,

$$\sin \omega t = \sin \pi = 0, \text{ so that } \mathcal{E} = 0.$$

4. When $\omega t = \frac{3\pi}{2}$ the plane of the coil is again parallel to $\vec{B}$,

$$\sin \omega t = \sin \frac{3\pi}{2} = -1, \quad \text{so that } \mathcal{E} = -\mathcal{E}_0$$

5. When $\omega t = 2\pi$, the plane of the coil again becomes perpendicular to $\vec{B}$ after completing one rotation,

$$\sin \omega t = \sin 2\pi = 0, \quad \text{so that } \mathcal{E} = 0$$

As the coil continues to rotate in the same sense, the same cycle of changes repeats again and again. As shown in Fig. 6.18(*b*), the graph between emf $\mathcal{E}$ and time t is a sine curve. Such an emf is called *sinusoidal or alternating emf.* Both the magnitude and direction of this emf change regularly with time.

The fact that an induced emf is set up in a coil when rotated in a magnetic field forms the basic principle of a dynamo or a generator.

For Your Knowledge

- The magnetic flux linked with a surface is maximum when it is held perpendicular to the direction of the magnetic field and the flux linked is zero when the surface is held parallel to the direction of the magnetic field.
- Induced emf is set up whenever the magnetic flux linked with a circuit changes even if the circuit is open. However, the induced current flows only when the circuit is closed.
- No emf is induced when a coil and a magnet move with the same velocity in the same direction.
- No emf is induced when a magnet is rotated about its own axis. However, emf is induced when a magnet is rotated about an axis perpendicular to its length.
- No emf is induced when a closed loop moves totally inside a uniform magnetic field.
- Just as a changing magnetic field produces an electric field, a changing electric field also sets up a magnetic field.
- The electric fields created by stationary charges have vanishing and path independent loop integrals. Such fields are called *conservative fields.*

  $$\oint \vec{E} \cdot d\vec{l} = 0$$

- The electric fields created by time-varying magnetic fields have non-vanishing loop integrals and are called *non-conservative fields.* Their loop integrals are path dependent.

  $$\oint \vec{E} \cdot d\vec{l} = \frac{d\phi}{dt}$$

- Electric potential is meaningful only for electric fields produced by stationary charges. It has no meaning for electric fields set up by magnetic induction.
- The heart beating induces a.c. in the surrounding tissues. The detection and study of these currents is called *electrocardiography* which provides valuable information regarding the pathology of the heart.
- **Migration of birds.** Every winter birds from Siberia fly unerringly to water spots in the Indian subcontinent. It is believed that migratory birds make use of earth's magnetic field to determine their direction. As birds contain no ferromagnetic material, so electromagnetic induction appears to be the only mechanism to determine direction. However, very small emfs induced across the bodies of these birds create a doubt about the validity of this hypothesis. So the migration pattern of birds is still a mystery.

Examples based on Induced EMF in a Rotating Coil

Formulae Used

1. $\mathcal{E} = \mathcal{E}_0 \sin \omega t$
2. $\mathcal{E}_0 = NBA\omega$, where $\omega = 2\pi f$
3. Maximum induced current, $I_0 = \frac{\mathcal{E}_0}{R}$.

Units Used

The induced emf $\mathcal{E}$ and maximum induced emf $\mathcal{E}_0$ are in volt, field B in tesla, area A in m^2, angular frequency ω in $rad s^{-1}$, current I_0 in ampere, resistance R in ohm.

Example 25. *A circular coil of area 300 cm^2 and 25 turns rotates about its vertical diameter with an angular speed of 40 s^{-1} in a uniform horizontal magnetic field of magnitude 0.05 T. Obtain the maximum voltage induced in the coil.* [NCERT]

Solution. Here $A = 300\ cm^2 = 300 \times 10^{-4} m^2$, N = 25, $\omega = 40\ s^{-1}$, $B = 0.05$ T

The maximum voltage induced in the coil is

$$\mathcal{E}_0 = NBA\omega = 25 \times 0.05 \times 300 \times 10^{-4} \times 40 = \mathbf{1.5\ V.}$$

Example 26. *A flat coil of 500 turns, each of area $5 \times 10^{-3}\ m^2$, rotates in a uniform magnetic field of 0.14 T at an angular speed of 150 rad s^{-1}. The coil has a resistance of 5 Ω. The induced emf is applied to an external resistance of 10 Ω. Calculate the peak current through the resistance.*

Solution. Here $A = 5 \times 10^{-3} m^2$, $B = 0.14$ T, $\omega = 150$ rad s^{-1}

Total resistance, $R = 5 + 10 = 15\ \Omega$

$$\mathcal{E}_0 = NBA\omega$$
$$= 500 \times 0.14 \times 5 \times 10^{-3} \times 150 = 52.5\ V$$

Peak current, $I_0 = \frac{\mathcal{E}_0}{R} = \frac{52.5}{15} = \mathbf{3.5\ A.}$

Example 27. *An athlete peddles a stationary tricycle whose pedals are attached to a coil having 100 turns each of area 0.1 m^2. The coil, lying in the X-Y plane is rotated, in this plane, at the rate of 50 rpm, about the Y-axis, in a region where a uniform magnetic field, $\vec{B} = (0.01)\hat{k}$ tesla, is present. Find the (i) maximum emf (ii) average emf, generated in the coil over one complete revolution.* [CBSE Sample Paper 13]

Solution. Here $N = 100$, $A = 0.1\ m^2$,

$$f = 50 \text{ rpm} = \frac{50}{60} \text{ rps} = \frac{5}{6} \text{ rps}$$

$$\vec{B} = (0.01)\hat{k} \text{ tesla} \Rightarrow B = 0.01\ T$$

(*i*) Maximum emf generated in the coil,

$$\mathcal{E}_0 = NBA\omega = NBA \times 2\pi f$$
$$= 100 \times 0.01 \times 0.1 \times 2 \times \pi \times \frac{5}{6} = \frac{\pi}{6} V \simeq \mathbf{0.52\ V.}$$

(*ii*) As the generated emf varies sinusoidally with time, so average emf generated in the coil over one complete cycle = 0.

Example 28. *A rectangular coil of wire has dimensions 0.2 m × 0.1 m. The coil has 2000 turns. The coil rotates in a magnetic field about an axis parallel to its length and perpendicular to the magnetic field of 0.02 Wb m^{-2}. The speed of rotation of the coil is 4200 rpm. Calculate (i) the maximum value of the induced emf in the coil (ii) the instantaneous value of induced emf when the plane of the coil has rotated through an angle of 30° from the initial position.*

Solution. Here $A = 0.2 \times 0.1\ m^2 = 2 \times 10^{-2}\ m^2$, $N = 2000$, $B = 0.02\ Wbm^{-2}$

$$f = 4200 \text{ rpm} = \frac{4200}{60} \text{ rps} = 70 \text{ rps}$$
$$\omega = 2\pi f = 2\pi \times 70 = 140\pi \text{ rad s}^{-1}$$

(*i*) The maximum value of the induced emf in the coil is

$$\mathcal{E}_0 = NBA\omega$$
$$= 2000 \times 0.02 \times 2 \times 10^{-2} \times 140 \times \frac{22}{7} V = \mathbf{3520\ V.}$$

(*ii*) The instantaneous value of induced emf when the coil has turned through 30° is

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$
$$= \mathcal{E}_0 \sin 30° = 3520 \times \frac{1}{2} V = \mathbf{1760\ V.}$$

Example 29. *A rectangular coil of 200 turns of wire, 15 cm × 40 cm makes 50 revolutions/second about an axis perpendicular to the magnetic field of 0.08 weber / m^2. What is the instantaneous value of induced emf when the plane of the coil makes an angle with the magnetic lines of (i) 0° (ii) 60° and (iii) 90° ?*

Solution. Here $N = 200$,

$$A = 15\ cm \times 40\ cm = 15 \times 40 \times 10^{-4}\ m^2 = 6 \times 10^{-2} m^2$$
$$B = 0.08\ Wb\ m^{-2},\ \omega = 2\pi \times 50 = 100\pi \text{ rad s}^{-1}$$

Induced emf at any instant is given by

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t = NBA\omega \sin \omega t$$

If the plane of the coil makes an angle α with the magnetic lines of force, then the angle between the normal to the plane of the coil and the magnetic field will be

$$\omega t = 90° - \alpha$$

(*i*) When $\alpha = 0°$, $\omega t = 90° - 0° = 90°$

$\therefore$ $\mathcal{E} = NBA\omega \sin 90°$

$$= 200 \times 0.08 \times 6 \times 10^{-2} \times 100\pi \times 1\ V = \mathbf{301.6\ V.}$$

(*ii*) When $\alpha = 60°$, $\omega t = 90° - 60° = 30°$

$\therefore \quad \mathcal{E} = NBA\omega \sin 30° = 301.6 \times \frac{1}{2} = \mathbf{150.8\ V.}$

(*iii*) When $\alpha = 90°$, $\omega t = 90° - 90° = 0°$

$\therefore \quad \mathcal{E} = NBA\omega \sin 0° = \mathbf{0.}$

Problems For Practice

1. A closely wound rectangular coil of 200 turns and size 0.3 m × 0.1 m is rotating in a magnetic field of induction 0.005 Wb m^{-2} with a frequency of revolution 1800 rpm about an axis normal to the field. Calculate the maximum value of induced emf. (Ans. 5.65 V)
2. A rectangular coil of dimensions 0.1 m × 0.5 m consisting of 2000 turns rotates about an axis parallel to its longer side, making 2100 revolutions per minute in a field of 0.1 T. What is the maximum emf induced in the coil ? Also find the instantaneous emf, when the coil is 60° to the field. (Ans. 2200 V, 1100 V)
3. The armature coil of a generator has 20 turns and its area is 0.127 m^2. How fast should it be rotated in a magnetic field of 0.2 Wbm^{-2}, so that the peak value of induced emf is 160 V ? (Ans. 50 rps)
4. A 50 turn coil of area 500 cm^2 is rotating at a rate of 50 rounds per second perpendicular to a magnetic field of 0.5 Wbm^{-2}. Calculate the maximum value of induced emf. (Ans. 392.5 V)
5. Calculate the maximum emf induced in a coil of 100 turns and 0.01 m^2 area rotating at the rate of 50 rps about an axis perpendicular to a uniform magnetic field of 0.05 T. If the resistance of the coil is 30 Ω, what is the maximum power generated by it ? (Ans. 15.7 V, 8.23 W)

HINTS

1. $\mathcal{E}_0 = NBA \times 2\pi f$

$$= 200 \times 0.005 \times 0.3 \times 0.1 \times 2 \times 3.14 \times \frac{1800}{60} = \mathbf{5.65\ V.}$$

2. $\mathcal{E}_0 = NBA \times 2\pi f$

$$= 2000 \times 0.1 \times 0.1 \times 0.5 \times 2 \times \frac{22}{7} \times \frac{2100}{60} = \mathbf{2200\ V.}$$

Here $\omega t = 90° - 60° = 30°$

$\therefore \quad \mathcal{E} = \mathcal{E}_0 \sin 30° = 2200 \times \sin 30° = \mathbf{1100\ V.}$

3. Frequency, f

$$= \frac{\mathcal{E}_0}{2\pi\, NBA} = \frac{160}{2 \times 3.14 \times 20 \times 0.2 \times 0.127} \simeq \mathbf{50\ rps.}$$

4. Use $\mathcal{E}_0 = NBA \times 2\pi f$.
5. $\mathcal{E}_0 = NBA\omega = 100 \times 0.05 \times 0.01 \times 2\pi \times 50 = \mathbf{15.7\ V}$ $[\because \omega = 2\pi f]$

Maximum current, $I_0 = \frac{15.7}{30} = 0.524$ A

Power generated $= \mathcal{E}_0 I_0 = 15.7 \times 0.524 = \mathbf{8.23\ W.}$

6.11 EDDY CURRENTS

12. *What are eddy currents ? Give some experiments to demonstrate their existence. What is the effect of eddy currents in electrical appliances, where iron core is used ? How are eddy currents minimised ? Describe some of the important applications of eddy currents.*

Eddy currents. Currents can be induced, not only in conducting coils, but also in conducting sheets or blocks. Whenever the magnetic flux linked with a metal sheet or block changes, an emf is induced in it. The induced currents flow in closed paths in planes perpendicular to the lines of force throughout the body of the metal. These currents look like *eddies* or *whirl- pools* in water and so they are known as ***eddy currents***. As these currents were first discovered by *Focault* in 1895, so eddy currents are also known as ***Focault currents***.

Eddy currents are the currents induced in solid metallic masses when the magnetic flux threading through them changes.

Eddy currents also oppose the change in magnetic flux, so their direction is given by Lenz's law.

Experiments to demonstrate eddy currents :

Experiment 1. Take a pendulum having its bob in the form of a flat copper plate. As shown in Fig. 6.21(*a*), it is free to oscillate between the pole pieces of an electromagnet. In the absence of any magnetic field, the pendulum swings freely. As the electromagnet is switched on, the oscillations of the pendulum get highly damped and soon it comes to rest. This is because as the copper plate moves in between the pole pieces of the magnet, magnetic flux threading through it changes. So eddy currents are set up in it which according to Lenz's law, oppose the motion of the copper plate in the magnetic field. Eddy currents flow anticlockwise as the plate swings into the field and clockwise as the plate swings out of the field.

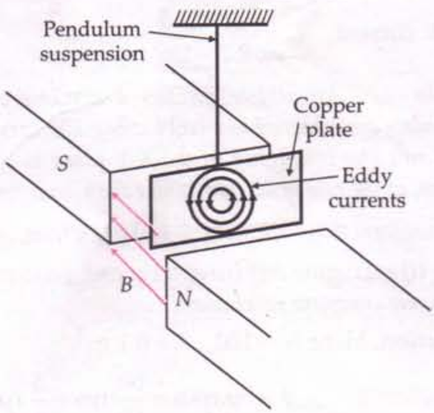


Fig. 6.21 (*a*) Eddy currents damp the oscillations of a copper plate in a magnetic field.

Experiment 2. Now take the pendulum of a flat copper plate with narrow slots cut across it, as shown in Fig 6.21(*b*). As the electromagnet is switched on, eddy currents are set up in the plate. But this plate swings for longer duration than the plate without slots. This is because the loop has much larger paths for the electrons to travel. Larger paths offer more resistance to electrons and so the eddy currents are sufficiently reduced. As a result, the opposition to the oscillations becomes very small.

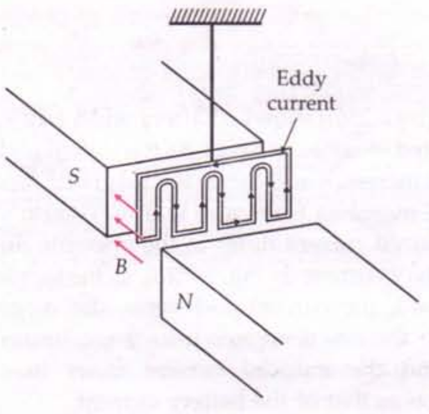


Fig. 6.21 (*b*) Reduced eddy currents in a slotted copper plate.

Experiment 3. Take a cylindrical electromagnet fed by an A.C. source and place a small metal disc over its top. As the current is switched on, the magnetic field at the disc rises from zero to a finite value, setting up eddy currents which effectively convert it into a small magnet. If initially the top end of the electromagnet acquires N-polarity, then by Lenz's law, the lower face of the small magnetic disc will also have N-polarity, resulting in a repulsive force. The disc is thus seen to be thrown up as the current in the electromagnet is switched on.

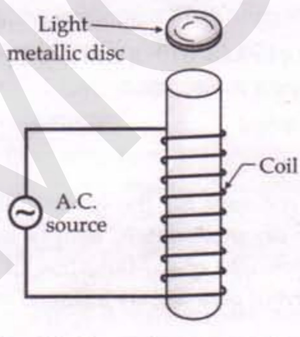


Fig. 6.22 A light metal disc on top of an electro-magnet is thrown up as the current is switched on.

Undesirable effects of eddy currents. Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos, and also in the cores of transformers, which experience flux changes, when they are in use. Eddy currents cause unnecessary heating and wastage of power. The heat produced by eddy currents may even damage the insulation of coils.

Minimisation. The eddy currents can be reduced by using *laminated core* which instead of a single solid mass consists of thin sheets of metal, insulated from each other by a thin layer of varnish, as shown in Fig. 6.23. The planes of the sheets are placed perpendicular to the direction of the currents that would be set up by the emf induced in the material. The insulation between the sheets then offers high resistance to the induced emf and the eddy currents are substantially reduced.

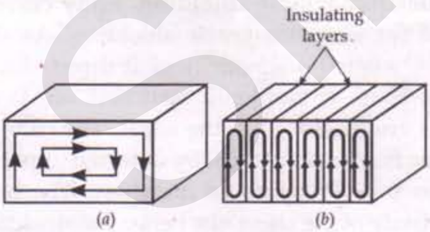


Fig. 6.23 (*a*) Solid core, (*b*) Laminated core.

Applications of eddy currents. Although eddy currents are undesirable, still they find applications in the following devices :

1. Induction furnace. If a metal specimen is placed in a rapidly changing magnetic field (produced by high frequency a.c.), very large eddy currents are set up. The heat produced is sufficient to even melt the metal. This process is used in the extraction of some metals from their ores.

2. Electromagnetic damping. When a current is passed through a galvanometer, its coil suffers few oscillations before coming to rest in the final position. As the coil moves in the magnetic field, induced current is set in the coil which opposes its motion. The oscillations of the coil are damped. This is called *electromagnetic damping*. The electromagnetic damping can be further increased by winding the coil on a light copper or aluminium frame. As the frame moves in the magnetic field, eddy currents are set up in the frame which resist the motion of the coil. This is how a galvanometer is rendered *dead beat, i.e.*, the coil does not oscillate – it deflects and stays in the final position immediately.

3. Electric brakes. A strong magnetic field is applied to the rotating drum attached to the wheel. Eddy currents set up in the drum exert a torque on the drum so as to stop the train.

4. Speedometers. In a speedometer, a magnet rotates with the speed of the vehicle. The magnet is placed inside an aluminium drum which is carefully pivoted and held in position by a hair spring. As the magnet rotates, eddy currents are set up in the drum which oppose the motion of the magnet. A torque is exerted on the drum in the opposite direction which deflects the drum through an angle depending on the speed of the vehicle.

5. Induction motor. In an a.c. induction motor, a rotating magnetic field is produced by two single phase alternating currents having a phase difference of 90°. A metallic rotor is placed in the magnetic field. The eddy currents set up in the rotor tend to oppose the relative motion between the rotating magnetic field and the rotor. As a result, the rotor also starts rotating about its axis.

6. Electromagnetic shielding. Eddy currents may be used for electromagnetic shielding. As shown in Fig. 6.24, when a magnetic field B, directed towards a metallic sheet is suddenly switched on, large eddy currents are produced in the sheet. The change in the magnetic field is only partially detected at points (such as P) on the other side of the sheet. The higher the conductivity of the sheet, the better the shielding of the transient magnetic field.

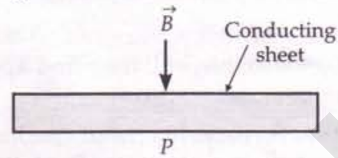


Fig. 6.24 Electromagnetic shielding.

7. Inductothermy. Eddy currents can be used to heat localised tissues of the human body. This branch is called inductothermy.

8. Energy meters. In energy meters used for measuring electric energy, the eddy currents induced in an aluminium disc are made use of.

For Your Knowledge

- Eddy currents are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes.
- Eddy currents tend to follow the path of least resistance inside a conductor. So they form irregularly shaped loops. However, their directions are not random, but guided by Lenz's law.
- Eddy currents have both undesirable effects and practically useful applications.
- Eddy currents can be induced in biological tissues. For example, the cavity of the eye is filled with a conducting fluid. A large transient magnetic field of 1 T alternating at a frequency of 60 Hz then induces such a large current in the retina that it produces a *sensation of intense brightness.*

6.12 SELF-INDUCTION

13. *What is meant by self-induction ? Define self-inductance. Give its units and dimensions.*

Self-induction. When a current flows in a coil, it gives rise to a magnetic flux through the coil itself. As the strength of current changes, the linked magnetic flux changes and an opposing emf is induced in the coil. This emf is called ***self-induced emf*** or ***back emf*** and the phenomenon is known as ***self-induction***.

Self-induction is the phenomenon of production of induced emf in a coil when a changing current passes through it.

Figure 6.25(a) shows a battery and a tapping switch connected in series to a coil. As the switch is closed, the current increases and hence the magnetic flux through the coil increases from zero to a maximum value and the induced current flows in the opposite direction of the battery current. In Fig. 6.25(b), as the tapping switch is opened, the current and hence the magnetic flux through the coil decreases from a maximum value to zero and the induced current flows in the same direction as that of the battery current.

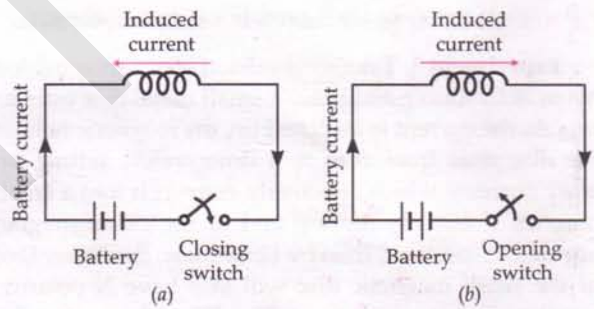


Fig. 6.25 Induced current in a coil when the circuit is (a) closed and (b) opened.

Coefficient of self-induction. At any instant, the magnetic flux ϕ linked with a coil is proportional to the current I through it, *i.e.*,

$$\phi \propto I$$

or $$\phi = LI \quad ...(1)$$

where L is a constant for the given coil and is called *self-inductance* or, more often, simply *inductance*. It is also called *coefficient of self-induction* of the coil. Any change in current sets up an induced emf in the coil given by

$$\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \quad ...(2)$$

If in equation (1), $I = 1$, then $\phi = L$

Thus the self-inductance of a coil is numerically equal to the magnetic flux linked with the coil when a unit current flows through it.

Again from equation (2), if $\frac{dI}{dt} = 1$, then $\varepsilon = -L$

Thus the self-inductance of a coil may be defined as the induced emf set up in the coil due to a unit rate of change of current through it.

Units of self-inductance. From equation (2),

$$L = \frac{\varepsilon}{dI/dt}$$

$\therefore$ SI unit of $L = \frac{1\text{ V}}{1\text{ As}^{-1}} = 1\text{ VsA}^{-1} = 1$ henry (H)

The *self-inductance of a coil is said to be one* ***henry*** *if an induced emf of one volt is set up in it when the current in it changes at the rate of one ampere per second.*

From equation (1), one may note that self-inductance is the ratio of magnetic flux and current. So its SI unit is *weber per ampere*. Hence

$$1\text{ henry (H)} = 1\text{ VsA}^{-1} = 1\text{ WbA}^{-1}$$

Dimensions of self-inductance. We know that

$$L = \frac{\phi}{I} = \frac{BA}{I} = \frac{F}{qv\sin\theta} \cdot \frac{A}{I} \qquad [\because F = qvB\sin\theta]$$

$\therefore$ Dimensions of L

$$= \frac{\text{MLT}^{-2}.\text{L}^2}{\text{C.LT}^{-1}.\text{A}} = \frac{\text{ML}^2\text{T}^{-2}}{\text{A.A}} = [\textbf{ML}^2\textbf{T}^{-2}\textbf{A}^{-2}]. \qquad [1\text{ CT}^{-1} = 1\text{ A}]$$

14. *Describe an experiment to demonstrate the phenomenon of self-induction.*

Experiment to demonstrate self-induction. Take a solenoid having a large number of turns of insulated wire wound over a soft iron core. Such a solenoid is called a choke coil. Connect the solenoid in series with a battery, a rheostat and a tapping key. Connect a 6 V bulb in parallel with the solenoid. Press the tapping key and adjust the current with the help of rheostat so that the bulb just glows faintly. As the tapping key is released, the bulb glows brightly for a moment and then goes out. This is because as the circuit is broken suddenly, the magnetic flux linked with the coil suddenly vanishes, *i.e.*, the rate of change of magnetic flux linked with the coil is very large. Hence large self-induced emf and current are produced in the coil which make the bulb glow brightly for a moment.

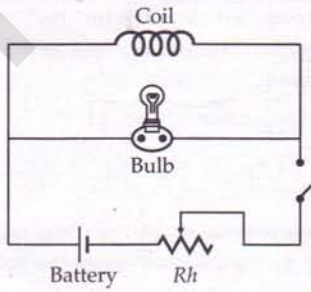


Fig. 6.26 Demonstration of self-induction.

6.13 SELF-INDUCTANCE OF A LONG SOLENOID

15. *Derive an expression for the self-inductance of a long solenoid. State the factors on which the self-inductance of a coil depends.*

Self-inductance of a long solenoid. Consider a long solenoid of length l and radius r with $r << l$ and having n turns per unit length. If a current I flows through the coil, then the magnetic field inside the coil is almost constant and is given by

$$B = \mu_0 nI$$

Magnetic flux linked with each turn

$$= BA = \mu_0 nIA$$

where $A = \pi r^2$ = the cross-sectional area of the solenoid.

$\therefore$ Magnetic flux linked with the entire solenoid is

ϕ = Flux linked with each turn $\times$ total number of turns

$= \mu_0 nIA \times nl = \mu_0 n^2 lAI$

But $\phi = LI$

$\therefore$ Self-inductance of the long solenoid is

$$L = \mu_0 n^2 lA$$

If N is the total number of turns in the solenoid, then $n = N/l$ and so

$$L = \frac{\mu_0 N^2 A}{l}$$

If the coil is wound over a material of high relative magnetic permeability μ_r (*e.g.*, soft iron), then

$$L = \mu_r \mu_0 n^2 lA = \frac{\mu_r \mu_0 N^2 A}{l}.$$

Factors on which self-inductance depends. Obviously, the self-inductance of a solenoid depends on its geometry and magnetic permeability of the core material.

1. *Number of turns.* Larger the number of turns in the solenoid, larger is its self-inductance.

$$L \propto N^2$$

2. *Area of cross-section.* Larger the area of cross-section of the solenoid, larger is its self-inductance.

$$L \propto A$$

3. *Permeability of the core material.* The self inductance of a solenoid increases μ_r times if it is wound over an iron core of relative permeability μ_r.

6.14 PHENOMENA ASSOCIATED WITH SELF-INDUCTION

16. *Describe some phenomena associated with self-induction.*

Phenomena associated with self-induction.

1. ***Sparking.*** The break of a circuit is very sudden. When the circuit is switched off, a large self induced emf is set up in the circuit in the same direction as the original emf. This causes a big spark across the switch.

2. ***Non-inductive winding.*** In resistance boxes and post office boxes, different resistance coils have to be used. Here the wire is first doubled over itself and then wound in the form of a coil over a bobbin. Due to this, the currents in the two halves of the wire flow in opposite directions as shown in Fig. 6.27. The inductive effects of the two halves of the wire, being in opposite directions, cancel each other. The net self-inductance of the coil is minimum. Such a winding of coils is called *non-inductive winding*. The resistance coils having no self-inductance are called *non-inductive resistances.*

Fig. 6.27 Non-inductive winding of resistance coil.

3. ***Electromagnetic damping.*** Refer to applications of eddy currents in section 6.11.

For Your Knowledge

- Inductance is a measure of the ratio of induced flux ϕ to the current I. It is a scalar quantity having the dimensions of magnetic flux divided by current. Its dimensions in terms of the fundamental quantities are $[ML^2T^{-2}A^{-2}]$. Its SI unit is WbA^{-1} or VsA^{-1} which is called *henry* (H). It is named in honour of *Joseph Henry* who discovered electromagnetic induction in USA independently of *Faraday* in England.
- Inductance plays the role of electrical inertia. The analogue of self-inductance in mechanics is mass.
- A solenoid made from a thick wire has a negligible resistance but a sufficiently large self-inductance. Such an element is called ***ideal inductor***, denoted by -ooo-.
- A wire itself cannot act as an inductor because the magnetic flux linked with the wire of negligible cross-sectional area is zero. Only a wire bent into the form of a coil can act as an inductor. Moreover, the self-induced emf appears only during the time the current through it is changing.
- The inductance of a coil depends on its geometry and the intrinsic properties of the material that fills up the space inside it. In this sense, it bears similarity to capacitance and resistance. The capacitance of a parallel plate capacitor depends on the plate area and plate separation (geometry) and the dielectric constant κ of the interposing medium (intrinsic material property). Similarly, the resistance of a conductor depends on its length and cross-sectional area (geometry) and resistivity (intrinsic material property).
- The capacitance, resistance, inductance and diode (described in chapter 14) constitute the ***four passive elements*** of an electrical circuit. In fact, these are the ***four alphabets*** of electrical/electronic engineering.

6.15 MUTUAL INDUCTION

17. *What is meant by mutual induction ? Define the term mutual inductance. Give its units and dimensions.*

Mutual induction. *Mutual induction is the phenomenon of production of induced emf in one coil due to a change of current in the neighbouring coil.*

As shown in Fig. 6.28, consider two coils P and S placed close to each other. The coil P is connected in series to a battery B and a rheostat Rh through a tapping key K. The coil S is connected to a galvanometer G. When a current flows through coil P, it produces a magnetic field which produces a magnetic flux through coil S. If the current in the coil P is varied, the magnetic flux linked with the coil S changes which induces an emf and hence a current in it, as is seen from the deflection in the galvanometer. The coil P is called the *primary coil* and coil S, the *secondary coil*, because it is the former which causes an induced emf in the latter.

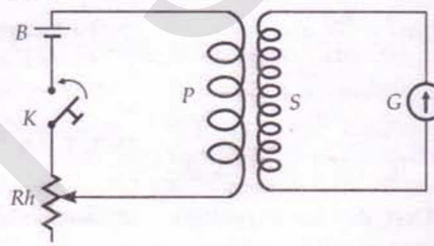


Fig. 6.28 Mutual induction.

Coefficient of mutual induction. At any instant,

Magnetic flux linked with the secondary coil
$\propto$ current in the primary coil

i.e. $\phi \propto I$

or $$\phi = MI \quad ...(1)$$

The proportionality constant M is called *the mutual inductance* or *coefficient of mutual induction* of the two coils. Any change in the current I sets up an induced emf in the secondary coil which is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\,.\frac{dI}{dt} \quad ...(2)$$

If in equation (1), $I = 1$, then $\phi = M$

Thus the mutual inductance of two coils is numerically equal to the magnetic flux linked with one coil when a unit current passes through the other coil.

Again, from equation (2), if

$$\frac{dI}{dt} = 1, \text{ then } \varepsilon = -M$$

The mutual inductance of two coils may be defined as the induced emf set up in one coil when the current in the neighbouring coil changes at the unit rate.

Unit of mutual inductance. From equation (2), we have

$$M = \frac{\mathcal{E}}{\frac{dI}{dt}}$$

$\therefore$ SI unit of $M = \frac{1\ \text{V}}{1\ \text{As}^{-1}} = 1\ \text{VsA}^{-1} = 1$ henry(H)

The mutual inductance of two coils is said to be one ***henry*** *if an induced emf of one volt is set up in one coil when the current in the neighbouring coil changes at the rate of 1 ampere per second.*

6.16 MUTUAL INDUCTANCE OF TWO LONG SOLENOIDS

18. *Derive an expression for the mutual inductance of two long coaxial solenoids. State the factors on which mutual inductance depends. What is coefficient of coupling ?*

Mutual inductance of two long solenoids. As shown in Fig. 6.29, consider two long co-axial solenoids S_1 and S_2, with S_2 wound over S_1.

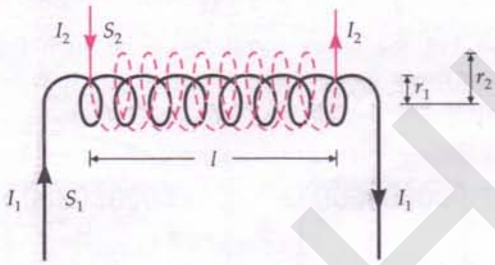


Fig. 6.29 Two long coaxial solenoids of same length l.

Let l = length of each solenoid

r_1, r_2 = radii of the two solenoids

$A = \pi r_1^2$

= area of cross-section of inner solenoid S_1

N_1, N_2 = number of turns in the two solenoids

First we pass a time varying current I_2 through S_2. The magnet field set up inside S_2 due to I_2 is

$$B_2 = \mu_0 n_2 I_2$$

where $n_2 = N_2 / l$ = the number of turns per unit length of S_2.

Total magnetic flux linked with the inner solenoid S_1 is

$$\phi_1 = B_2 A N_1 = \mu_0 n_2 I_2 \cdot A N_1$$

$\therefore$ Mutual inductance of coil 1 with respect to coil 2 is

$$M_{12} = \frac{\phi_1}{I_2} = \mu_0 n_2 A N_1 = \frac{\mu_0 N_1 N_2 A}{l}$$

We now consider the flux linked with the outer solenoid S_2 due to the current I_1 in the inner solenoid S_1. The field B_1 due to I_1 is constant inside S_1 but zero in the annular region between the two solenoids. Hence

$$B_1 = \mu_0 n_1 I_1$$

where $n_1 = N_1 / l$ = the number of turns per unit length of S_1.

Total flux linked with the outer solenoid S_2 is

$$\phi_2 = B_1 A N_2 = \mu_0 n_1 I_1 \cdot A N_2 = \frac{\mu_0 N_1 N_2 A I_1}{l}$$

$\therefore$ Mutual inductance of coil 2 with respect to coil 1 is

$$M_{21} = \frac{\phi_2}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Clearly $M_{12} = M_{21} = M$ (say)

$$\therefore \quad M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 A l = \mu_0 n_1 n_2 \pi r_1^2 l$$

Thus, the mutual inductance of two coils is the property of their combination. It does not matter which one of them functions as the primary or the secondary coil. This fact is known as ***reciprocity theorem***.

Factors on which mutual inductance depends. The mutual inductance of two solenoids depends on their geometry and the magnetic permeability of the core material.

1. ***Number of turns.*** Larger the number of turns in the two solenoids, larger will be their mutual inductance.

$$M \propto N_1 N_2$$

2. ***Common cross-sectional area.*** Larger the common cross-sectional area of two solenoids, larger will be their mutual inductance.

3. ***Relative separation.*** Larger the distance between two solenoids, smaller will be the magnetic flux linked with the secondary coil due to current in the primary coil. Hence smaller will be the value of M.

4. ***Relative orientation of the two coils.*** M is maximum when the entire flux of the primary is linked with the secondary, *i.e.*, when the primary coil completely envelopes the secondary coil. M is minimum when the two coils are perpendicular to each other, as shown in Fig. 6.30.

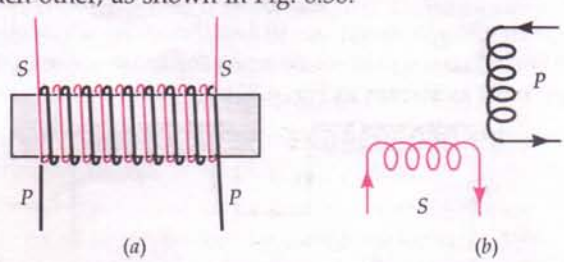


Fig. 6.30 (*a*) M is maximum when primary envelopes secondary, (*b*) M is minimum when primary is perpendicular to secondary.

5. ***Permeability of the core material.*** If the two coils are wound over an iron core of relative permeability μ_r, their mutual inductance increases μ_r times.

Coefficient of coupling. *The coefficient of coupling of two coils gives a measure of the manner in which the two coils are coupled together.* If L_1 and L_2 are the self- inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

The value of K lies between 0 and 1.

When the coupling is perfect *i.e.*, the entire flux of primary is linked with the secondary, M is maximum and $K = 1$.

When there is no coupling, $M = 0$ and $K = 0$.

Thus K is maximum for the coupling shown in Fig. 6.30(*a*) and minimum for the coupling shown in Fig. 6.30(*b*).

For Your Knowledge

- When two coils are inductively coupled, in addition to the emf produced due to mutual induction, induced emf is set up in the two coils due to self-induction also.
- The mutual inductance of two coils is a property of their combination. The value of M remains unchanged irrespective of the fact that current is passed through one coil or the other.
- While calculating the mutual inductance of two long co-axial solenoids, the cross-sectional area of the inner solenoid is to be considered.
- While calculating the mutual inductance of two co-axial solenoids of different lengths, the length of the larger solenoid is to be considered.

6.17 GROUPING OF INDUCTANCES*

19. *A circuit contains two inductors in series with self-inductances L_1 and L_2 and mutual inductance M. Obtain a formula for the equivalent inductance in the circuit.* [NCERT]

Inductances in series. (*i*) Let the series connection be such that the current flows in the same sense in the two coils as shown in Fig. 6.31(*a*).

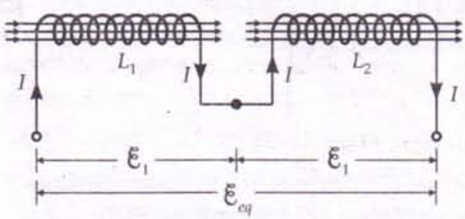


Fig. 6.31 (*a*) Inductance in series when fluxes get added.

Let L_{eq} be the equivalent inductance of the two self-inductances L_1 and L_2 connected in series. For the series combination, the emfs induced in the two coils get added up. Thus

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$$

If the rate of change of current in the series circuit is $\frac{dI}{dt}$, then

$$\varepsilon_1 = -L_1 \frac{dI}{dt} - M\frac{dI}{dt},$$

$$\varepsilon_2 = -L_2 \frac{dI}{dt} - M\frac{dI}{dt}$$

and $$\varepsilon_{eq} = -L_{eq}\frac{dI}{dt}$$

The negative sign throughout indicates that both self and mutual induced emfs are opposing the applied emf. Using the above equations, we have

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$$

or $$-L_{eq}\cdot\frac{dI}{dt} = -(L_1 + M + L_2 + M)\frac{dI}{dt}$$

or $$L_{eq} = L_1 + L_2 + 2M.$$

(*ii*) Let the series combination be such that the current flows in opposite senses in the two coils, as shown in Fig. 6.31(*b*).

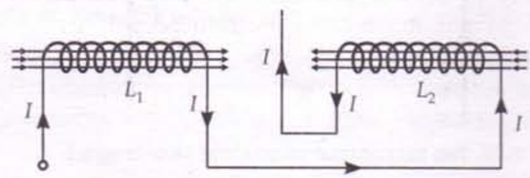


Fig. 6.31 (*b*) Inductances in series when fluxes get subtracted.

The emfs induced in the two coils will be

$$\varepsilon_1 = -L_1\frac{dI}{dt} + M\frac{dI}{dt},$$

$$\varepsilon_2 = -L_2\frac{dI}{dt} + M\frac{dI}{dt}$$

Here the mutual emfs act in the direction of applied emf and hence positive. For this series combination also, the emfs induced in the two coils get added up.

Hence $$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 = -[L_1 - M + L_2 - M]\frac{dI}{dt}$$

But $$\varepsilon_{eq} = -L_{eq}\frac{dI}{dt}$$

$$\therefore \quad -L_{eq}\frac{dI}{dt} = -[L_1 + L_2 - 2M]\frac{dI}{dt}$$

or $$L_{eq} = L_1 + L_2 - 2M.$$

20. *Two inductors of self-inductances L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. What is the equivalent inductance of the combination ?* [NCERT]

Inductances in parallel. For the parallel combination, the total current I divides up through the two coils as

$$I = I_1 + I_2$$

$$\therefore \quad \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

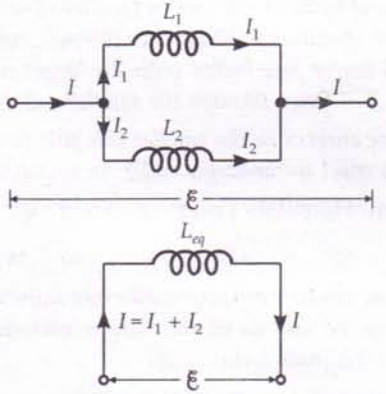


Fig. 6.32 Inductances in parallel

For parallel combination, induced emf across the combination is equal to the induced emf across each inductance. Thus

$$\mathcal{E} = -L_1 \frac{dI_1}{dt} \quad \text{or} \quad \frac{\mathcal{E}}{L_1} = -\frac{dI_1}{dt}$$

$$\mathcal{E} = -L_2 \frac{dI_2}{dt} \quad \text{or} \quad \frac{\mathcal{E}}{L_2} = -\frac{dI_2}{dt}$$

This is because the mutual inductance M is negligible. If L_{eq} is the equivalent inductance of the parallel combination, then

$$\mathcal{E} = -L_{eq} \cdot \frac{dI}{dt} = -L_{eq}\left[\frac{dI_1}{dt} + \frac{dI_2}{dt}\right]$$

$$= L_{eq}\left[-\frac{dI_1}{dt} - \frac{dI_2}{dt}\right]$$

or $$\frac{\mathcal{E}}{L_{eq}} = \left[\frac{\mathcal{E}}{L_1} + \frac{\mathcal{E}}{L_2}\right] \quad \text{or} \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

or $$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

If there is any mutual inductance M between the coils, then

$$L = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm M}.$$

Examples based on

Self-Induction and Mutual Induction

Formulae Used

1. For self-induction, $\phi = LI$
2. Self induced emf, $\mathcal{E} = -L\frac{dI}{dt}$
3. For Mutual induction, $\phi = MI$
4. Mutual induced emf, $\mathcal{E} = -M\frac{dI}{dt}$
5. Self-inductance of long solenoid,
$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al, \quad \text{where } n = \frac{N}{l}$$
6. Mutual inductance of two closely wound solenoids,
$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 Al,$$
where $n_1 = \frac{N_1}{l}, n_2 = \frac{N_2}{l}$

Units Used

Flux ϕ is in weber, inductances L and M in henry, emf $\mathcal{E}$ in volt, current I in ampere, cross-sectional area A in m^2, number of turns per unit length n, n_1 and n_2 are in m^{-1}.

Example 30. *What is the self-inductance of a coil, in which magnetic flux of 40 milliweber is produced when 2 A current flows through it ?* [CBSE F 02]

Solution. Here $\phi = 40$ mWb $= 40 \times 10^{-3}$ Wb, $I = 2$ A

Self-inductance,

$$L = \frac{\phi}{I} = \frac{40 \times 10^{-3}}{2} = \mathbf{2 \times 10^{-2}\ H.}$$

Example 31. *A 200 turn coil of self-inductance 20 mH carries a current of 4 mA. Find the magnetic flux linked with each turn of the coil.*

Solution. Let ϕ be the magnetic flux linked with each of the N turns of the coil. Then

$$N\phi \propto I \quad \text{or} \quad N\phi = LI$$

$$\therefore \quad \phi = \frac{LI}{N} = \frac{20 \times 10^{-3} \times 4 \times 10^{-3}}{200}$$

$$= \mathbf{4 \times 10^{-7}\ Wb.}$$

Example 32. *If a rate of change of current of 4 As^{-1} induces an emf of 20 mV in a solenoid, what is the self-inductance of the solenoid ?* [CBSE D 96]

Solution. Here $\frac{dI}{dt} = 4\ As^{-1}$,

$$|\mathcal{E}| = 20\ mV = 20 \times 10^{-3}\ V$$

As $|\varepsilon| = L\dfrac{dI}{dt}$

$\therefore$ $$L = \frac{|\varepsilon|}{dI/dt} = \frac{20\times10^{-3}}{4}$$

$$= 5\times10^{-3}\text{ H} = \mathbf{5\ mH.}$$

Example 33. *A 12 V battery connected to a 6 Ω, 10 H coil through a switch drives a constant current through the circuit. The switch is suddenly opened. If it takes 1 ms to open the switch, find the average emf induced across the coil.*

Solution. Steady-state current $= \dfrac{12\text{ V}}{6\,\Omega} = 2$ A

Final current = 0

$\therefore$ $$\frac{dI}{dt} = \frac{(0-2)\text{ A}}{1\text{ ms}} = -2\times10^{-3}\text{ As}^{-1}$$

Induced emf,

$$\varepsilon = -L\frac{dI}{dt} = -10\times(-2\times10^{3}) = \mathbf{20{,}000\ V}$$

Such a high emf usually causes sparks across the open switch.

Example 34. *An inductor of 5 H inductance carries a steady current of 2 A. How can a 50 V self-induced emf be made to appear in the inductor ?* [Punjab 01]

Solution. Suppose the current reduces to zero in time t second. Thus

$L = 5$ H, $dI = 0 - 2 = -2$ A, $\varepsilon = 50$ V

As $\varepsilon = -L\dfrac{dI}{dt}$

$\therefore$ $50 = -5\times\dfrac{-2}{t}$ or $t = 0.2$ s

Hence an induced emf of 50 V can be generated by reducing the current to zero in 0.2 s.

Example 35. *What is the self-inductance of an air core solenoid 50 cm long and 2 cm radius if it has 500 turns ?*

Solution. Here $l = 50$ cm $= 0.50$ m,

$r = 2$ cm $= 2\times10^{-2}$ m, $N = 500$, $\mu_0 = 4\pi\times10^{-7}$ TmA^{-1}

Self-inductance of the solenoid is

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}$$

$$= \frac{4\pi\times10^{-7}\times(500)^2\times\pi\times(2\times10^{-2})^2}{0.50}$$

$$= \mathbf{7.89\times10^{-4}\ H.}$$

Example 36. *An air-cored solenoid with length 30 cm, area of cross-section 25 cm² and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit ?* [NCERT]

Solution. $l = 30$ cm $= 0.30$ m,

$A = 25\text{ cm}^2 = 25\times10^{-4}\text{ m}^2$, $N = 500$, $dt = 10^{-3}$ s,

$dI = 0 - 2.5 = -2.5$ A

$$\text{Back emf} = -L\frac{dI}{dt} = -\frac{\mu_0 N^2 A}{l}\cdot\frac{dI}{dt}$$

$$= -\frac{4\pi\times10^{-7}\times(500)^2\times2.5\times10^{-4}\times(-2.5)}{0.30\times10^{-3}}$$

$$= \mathbf{6.542\ V.}$$

Example 37. *A large circular coil, of radius R and a small circular coil, of radius r, are put in vicinity of each other. If the coefficient of mutual induction, for this pair, equals 1 mH, what would be the flux linked with the larger coil when a current of 0.5 A flows through the smaller coil ?*

When the current in the smaller coil falls to zero, what would be its effect in the larger coil ? [CBSE D 08C]

Solution. Here $M = 1$ mH, $I = 0.5$ A

Flux, $\phi = MI = 10^{-3}\text{H}\times0.5\text{ A} = \mathbf{5\times10^{-4}\ Wb.}$

When the current in the smaller coil falls to zero, an induced emf is set up in the larger coil due to the decrease in the linked flux.

Example 38. *What is the mutual inductance of a pair of coils if a current change of six ampere in one coil causes the flux in the second coil of 2000 turns to change by 12×10^{-4} Wb per turn ?*

Solution. Here $N = 2000$, $I = 6$ A

Flux per turn $= 12\times10^{-4}$ Wb

Total flux, $\phi = 2000\times12\times10^{-4} = 2.4$ Wb

As $\phi = MI$

$\therefore$ $$M = \frac{\phi}{I} = \frac{2.4}{6} = \mathbf{0.4\ H.}$$

Example 39. *An emf of 0.5 V is developed in the secondary coil, when current in primary coil changes from 5.0 A to 2.0 A in 300 millisecond. Calculate the mutual inductance of the two coils.* [ISCE 93]

Solution. Here $\varepsilon = 0.5$ V, $dI = 2 - 5 = -3$ A,

$dt = 300\text{ ms} = 300\times10^{-3}$ s

As $\varepsilon = -M\dfrac{dI}{dt}$

$\therefore$ $$0.5 = -M\times\frac{-3}{300\times10^{-3}}$$

or $M = \mathbf{0.05\ H.}$

Example 40. *If the current in the primary circuit of a pair of coils changes from 5 A to 1 A in 0.02 s, calculate (i) induced emf in the secondary coil if the mutual inductance between the two coils is 0.5 H and (ii) the change of flux per turn in the secondary, if it has 200 turns.*

Solution. (*i*) $\mathcal{E} = -M\frac{dI}{dt} = -0.5 \times \frac{(1-5)}{0.02} = \frac{2}{0.02}$

$= \mathbf{100\ V}.$

(*ii*) $\mathcal{E} = -N\frac{d\phi}{0.02}$

$\therefore$ Change in flux per turn,

$$d\phi = -\frac{100 \times 0.02}{200} = \mathbf{-0.01\ Wb.}$$

The negative sign shows a decrease of magnetic flux.

Example 41. *Over a solenoid of 50 cm length and 2 cm radius and having 500 turns, is wound another wire of 50 turns near the centre. Calculate the (i) mutual inductance of the two coils (ii) induced emf in the second coil when the current in the primary changes from 0 to 5 A in 0.02 s.*

Solution. Here $N_1 = 500$, $N_2 = 50$, $r = 2$ cm $= 0.02$ m $l = 50$ cm $= 0.50$ m, $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}

(*i*) The mutual inductance of the two coils is

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{\mu_0 N_1 N_2 . \pi r^2}{l}$$

$$M = \frac{4\pi \times 10^{-7} \times 500 \times 50 \times \pi \times (0.02)^2}{0.5}$$

$$= 78.96 \times 10^{-6}\ \text{H} = \mathbf{78.96\ \mu H}.$$

(*ii*) The emf induced in the second coil is

$$\mathcal{E} = -M\frac{dI}{dt} = -78.96 \times 10^{-6} . \frac{(5-0)}{0.02}$$

$$= -19.74 \times 10^{-3}\ \text{V} = \mathbf{-19.74\ mV}.$$

The negative sign indicates a back emf.

Example 42. *A solenoidal coil has 50 turns per centimeter along its length and a cross-sectional area of 4×10^{-4} m^2. 200 turns of another wire are wound round the first solenoid coaxially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils. Given $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}.* [ISCE 98]

Solution. Here $n_1 = 50$ turns per cm $= 5000$ turns per metre, $n_2 l = 200$ turns, $A = 4 \times 10^{-4}$ m^2

$$M = \mu_0 n_1 n_2 A l = \mu_0 n_1 (n_2 l) A$$

$$= 4\pi \times 10^{-7} \times 5000 \times 200 \times 4 \times 10^{-4}$$

$$= \mathbf{5.027 \times 10^{-4}\ H}.$$

Example 43. *A solenoid of length 50 cm with 20 turns per cm and area of cross-section 40 cm^2 completely surrounds another co-axial solenoid of the same length, area of cross-section 25 cm^2 with 25 turns per cm. Calculate the mutual-inductance of the system.* [NCERT]

Solution. Here $l = 50$ cm $= 0.50$ m

Total no. of turns in outer-solenoid,

$$N_1 = n_1 l = 20 \times 50 = 1000$$

Area of cross-section of outer solenoid,

$$A_1 = 40\ \text{cm}^2 = 40 \times 10^{-4}\ \text{m}^2$$

Total no. of turns in inner solenoid,

$$N_2 = n_2 l = 25 \times 50 = 1250$$

Area of cross-section of inner solenoid,

$$A_2 = 25\ \text{cm}^2 = 25 \times 10^{-4}\ \text{m}^2$$

To determine mutual inductance, we take area of cross-section of inner solenoid.

$$\therefore\ M = \frac{\mu_0 N_1 N_2 A_2}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times 1250 \times 25 \times 10^{-4}}{0.50}$$

$$= 7.85 \times 10^{-3}\ \text{H} = \mathbf{7.85\ mH}.$$

Example 44. *(a) A toroidal solenoid with an air-core has an average radius of 15 cm, area of cross-section 12 cm^2 and 1200 turns. Obtain the self-inductance of the toroid. Ignore field variation across the cross-section of the toroid.*

(b) A second coil of 300 turns is wound closely on the toroid above. If the current in the primary coil is increased from zero to 2.0 A in 0.05 s, obtain the induced emf in the second coil. [NCERT]

Solution. (*a*) The uniform magnetic field set up inside a solenoid is given by

$$B = \mu_0 n I = \frac{\mu_0 N}{2\pi r} . I \qquad \left[\because n = \frac{N}{2\pi r}\right]$$

$\therefore$ Total flux linked with the N turns is

$$\phi = NBA = N . \frac{\mu_0 N I}{2\pi r} . A = \frac{\mu_0 N^2 I A}{2\pi r}$$

$\therefore$ Self-inductance of the toroid is

$$L = \frac{\phi}{I} = \frac{\mu_0 N^2 A}{2\pi r}$$

$$= \frac{4\pi \times 10^{-7} \times (1200)^2 \times 12 \times 10^{-4}}{2\pi \times 0.15}$$

$$= 2.304 \times 10^{-3}\ \text{H} = \mathbf{2.3\ mH}.$$

(*b*) Here $N_1 = 1200$, $N_2 = 300$, $dt = 0.05$ s,

$dI = 2.0 - 0 = 2.0$ A

$\therefore$ The emf induced in the second coil is

$$\mathcal{E} = M\frac{dI}{dt} = \frac{\mu_0 N_1 N_2 A}{l}\frac{dI}{dt}$$

$$= \frac{4\pi \times 10^{-7} \times 1200 \times 300 \times 12 \times 10^{-4}}{2\pi \times 0.15} \times \frac{2.0}{0.05}$$

$$= \mathbf{0.023\ V}. \qquad [\because l = 2\pi r]$$

Example 45. *Fig. 6.33 shows a short solenoid of length 4 cm, radius 2.0 cm and number of turns 100 lying inside on the axis of a long solenoid, 80 cm length and number of turns 1500. What is the flux through the long solenoid if a current of 3.0 A flows through the short solenoid ? Also obtain the mutual inductance of the two solenoids.* [NCERT]

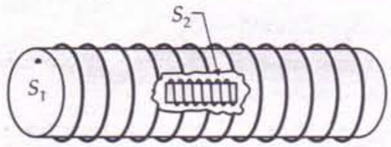


Fig. 6.33

Solution. As the short solenoid produces a complicated magnetic field, so it is difficult to calculate mutual inductance and flux through the outer solenoid. For this purpose we make use of the principle of reciprocity of mutual inductance *i.e.*,

$$M_{12} = M_{21}$$

Suppose S_1 represents the long solenoid and S_2, the short solenoid. Then

$$l_1 = 80 \text{ cm} = 0.80 \text{ m}, \quad N_1 = 1500$$
$$l_2 = 4 \text{ cm} = 0.04 \text{ m}, \quad N_2 = 100,$$
$$R_2 = 2.0 \text{ cm} = 0.02 \text{ m}, \quad I_2 = 3.0 \text{ A}$$

The uniform magnetic field inside the long solenoid is given by

$$B_1 = \frac{\mu_0 N_1 I_1}{l_1}$$

Since the short solenoid lies completely inside the long solenoid, the flux linked with it is given by

$$\phi_2 = N_2 A_2 B_1 = N_2 A_2 . \frac{\mu_0 N_1 I_1}{l_1}$$

$\therefore$ Flux through each turn of short solenoid

$$= \frac{\phi_2}{N_2} = \frac{\mu_0 N_1 I_1}{l_1} . A_2 = \frac{\mu_0 N_1 I_1}{l_1} . \pi R_2^2$$

By definition, $\phi_2 = M_{21} I_1$

or $$N_2 . \frac{\mu_0 N_1 I_1 . \pi R_2^2}{l_1} = M_{21} I_1$$

From the symmetry of mutual inductance, we have

$$M_{12} = M_{21} = \frac{\mu_0 \pi R_2^2 . N_1 N_2}{l_1}$$
$$= \frac{4\pi \times 10^{-7} \times \pi \times (0.02)^2 \times 1500 \times 100}{0.80} \text{ H}$$
$$= \mathbf{2.96 \times 10^{-4} \text{ H}}$$

The total flux linked with the long solenoid is

$$N_1 \phi_1 = M_{12} I_2 = 2.96 \times 10^{-4} \times 3.0 \text{ Wb}$$
$$= 8.88 \times 10^{-4} \text{ Wb} \simeq \mathbf{8.9 \times 10^{-4} \text{ Wb}}.$$

Example 46. *Three inductances are connected as shown Fig. 6.34 Find the equivalent inductance.* [Punjab 93]

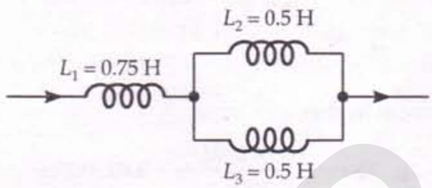


Fig. 6.34

Solution. The equivalent inductance L' of L_2 and L_3 is given by

$$\frac{1}{L'} = \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{0.5} + \frac{1}{0.5} = 4$$

or $$L' = \frac{1}{4} = 0.25 \text{ H}$$

Now L_1 and L' form a series combination, their equivalent inductance is given by

$$L = L_1 + L' = 0.75 + 0.25 = \mathbf{1 \text{ H}}.$$

Problems For Practice

1. Magnetic flux of 5 microweber is linked with a coil, when a current of 1 mA flows through it. What is the self-inductance of the coil ? **[Haryana 99]** (Ans. 5 mH)
2. Calculate the induced emf in a coil of 10 H inductance in which the current changes from 8 A to 3 A in 0.2 s. (Ans. 250 V)
3. A magnetic flux of 8×10^{-4} Wb is linked with each turn of a 200-turn coil when there is an electric current of 4 A in it. Calculate the self-inductance of the coil. (Ans. 4×10^{-2} H)
4. The self inductance of an inductor coil having 100 turns is 20 mH. Calculate the magnetic flux through the cross-section of the coil corresponding to a current of 4 mA. Also, find the total flux. **[CBSE PMT 2000]** (Ans. 8×10^{-7} Wb, 8×10^{-5} Wb)
5. A coil of inductance 0.5 H is connected to a 18 V battery. Calculate the rate of growth of current. (Ans. 36 As^{-1})
6. An average induced emf of 0.4 V appears in a coil when the current in it is changed from 10 A in one direction to 10 A in opposite in 0.40 second. Find the coefficient of self induction of the coil. **[CBSE Sample Paper 11]** (Ans. 8 mH)
7. A coil has a self-inductance of 10 mH. What is the maximum magnitude of the induced emf in the inductor, when a current $I = 0.1 \sin 200t$ ampere is sent through it. (Ans. 0.2 V)

8. What is the self-inductance of a solenoid of length 40 cm, area of cross-section 20 cm^2 and total number of turns 800 ? (Ans. 4.02 mH)

9. The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at the rate of 0.8 As^{-1}. Find the emf induced in it. (Ans. 6×10^{-4} V)

10. Calculate the mutual inductance between two coils when a current of 2 A changes to 6 A in 2 s and induces an emf of 20 mV in the secondary coil. **[Punjab 99]** (Ans. 10 mH)

11. The mutual inductance between two coils is 2.5 H. If the current in one coil is changed at the rate 2.0 As^{-1}, what will be the emf induced in the other coil ? (Ans. 5.0 V)

12. In a carspark coil, an emf of 40,000 V is induced in the secondary when the primary current changes from 4 A to zero in 10 μs. Find the mutual inductance between the primary and secondary windings of this spark coil. (Ans. 0.1 H)

13. If the current in the primary circuit of a pair of coils changes from 10 A to 0 in 0.1 s, calculate
    (i) the induced emf in the secondary if the mutual inductance between the two coils is 2 H, and
    (ii) the change of flux per turn in the secondary if it has 500 turns. (Ans. 200 V, – 0.04 Wb)

14. A conducting wire of 100 turns is wound over 1 cm near the centre of a solenoid of 100 cm length and 2 cm radius having 1000 turns. Calculate the mutual inductance of the two coils. **[Haryana 94]** (Ans. 1.58×10^{-4} H)

15. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely wound. If an initial current of 2 A is reversed in 0.25 s, find the emf induced in the coil. (Ans. 48 mV)

16. Calculate the mutual inductance between two coils if a current 10 A in the primary coil changes the flux by 500 Wb per turn in the secondary coil of 200 turns. Also determine the induced emf across the ends of the secondary coil in 0.5 s. (Ans. 10^4 H, 2×10^5 V)

HINTS

1. Here $\phi = 5\,\mu Wb = 5 \times 10^{-6}$ Wb ; $I = 1\,mA = 10^{-3}$ A
Self-inductance,
$$L = \frac{\phi}{I} = \frac{5 \times 10^{-6}}{10^{-3}} = 5 \times 10^{-3}\ H = \mathbf{5\ mH}$$

2. $\mathcal{E} = -L\frac{dI}{dt} = -L \cdot \frac{I_2 - I_1}{t} = -10 \times \frac{3-8}{0.2} = \mathbf{250\ V.}$

3. $N\phi = LI$
$$\therefore\ L = \frac{N\phi}{I} = \frac{200 \times 8 \times 10^{-4}}{4} = \mathbf{4 \times 10^{-2}\ H.}$$

4. Total magnetic flux,
$$N\phi = LI = 20 \times 10^{-3} \times 4 \times 10^{-3} = \mathbf{8 \times 10^{-5}\ Wb}$$
Magnetic flux through the cross-section of the coil,
$$\phi = \frac{8 \times 10^{-5}}{N} = \frac{8 \times 10^{-5}}{100} = \mathbf{8 \times 10^{-7}\ Wb.}$$

5. $\frac{dI}{dt} = \frac{\mathcal{E}}{L} = \frac{18}{0.5} = \mathbf{36\ As^{-1}}.$

6. $\mathcal{E} = -L\frac{I_2 - I_1}{t}$
$$0.4 = -L\left(\frac{-10-10}{0.40}\right) \text{ or } L = \frac{0.4}{50} = 0.008\ H = \mathbf{8\ mH.}$$

7. Here $L = 10\ mH = 10^{-2}\ H$, $I = 0.1 \sin 200t$
$$\mathcal{E} = L\frac{dI}{dt} = 10^{-2} \times \frac{d}{dt}(0.1 \sin 200t)$$
$$= 10^{-2} \times 0.1 \times 200 \cos 200t = 0.2 \cos 200t$$
Clearly, $\mathcal{E}$ will be maximum when $\cos 200t = 1$.
Therefore,
$$\mathcal{E}_{max} = 0.2 \times 1 = \mathbf{0.2\ V.}$$

8. Here $l = 40\ cm = 0.40$ m,
$A = 20\ cm^2 = 20 \times 10^{-4} m^2$,
$N = 800$, $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$
Self-inductance of the solenoid is
$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (800)^2 \times 20 \times 10^{-4}}{0.40}$$
$$= 4.02 \times 10^{-3}\ H = \mathbf{4.02\ mH.}$$

9. $$|\mathcal{E}| = \frac{LdI}{dt} = \frac{\mu_0 N^2 A}{l} \cdot \frac{dI}{dt}$$
$$= \frac{4\pi \times 10^{-7} \times (240)^2 \times \pi \times (0.02)^2}{0.12} \times 0.8$$
$$= \mathbf{6 \times 10^{-4}\ V.}$$

10. As $\mathcal{E} = -M\frac{dI}{dt}$
$$\therefore\ -20 \times 10^{-3} = -M\frac{6-2}{2} \quad \text{or} \quad M = \mathbf{10\ mH.}$$

11. $|\mathcal{E}| = M\frac{dI}{dt} = 2.5 \times 2.0 = \mathbf{5.0\ V.}$

12. $\mathcal{E} = -M\frac{dI}{dt}$
$$\therefore\ 40{,}000 = -M \times \frac{0-4}{10 \times 10^{-6}}$$
or $M = \mathbf{0.1\ H.}$

13. (i) $\mathcal{E} = -M\dfrac{dI}{dt} = -2 \times \dfrac{0-10}{0.1} = \mathbf{200\ V.}$

(ii) As $\mathcal{E} = -N\dfrac{d\phi}{dt} \quad \therefore \quad N\dfrac{d\phi}{dt} = M\dfrac{dI}{dt}$

or $\quad d\phi = \dfrac{M}{N} \cdot dI = \dfrac{2 \times (0-10)}{500} = \mathbf{-0.04\ Wb.}$

14. Here $N_1 = 1000, \quad l = 100\text{ cm} = 1\text{ m},$

$A = \pi r^2 = \pi(2 \times 10^{-2})^2\text{ m}^2, \quad N_2 = 100$

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 1000 \times 100 \times \pi \times 4 \times 10^{-4}}{1}$$

$$= 16\pi^2 \times 10^{-6} = 16 \times 9.87 \times 10^{-6}$$

$$= \mathbf{1.58 \times 10^{-4}\ V.}$$

15. $M = \dfrac{\mu_0 N_1 N_2 A}{l}$

$$= \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3}$$

$$= 3 \times 10^{-3}\text{ H}$$

$$\mathcal{E} = -M\frac{dI}{dt} = -3 \times 10^{-3}\left(\frac{-2-2}{0.25}\right)$$

$$= 48 \times 10^{-3}\text{V}$$

$$= \mathbf{48\ mV.}$$

16. As $N\phi = MI$

$\therefore \quad 200 \times 500 = M \times 10$

or $\quad M = \mathbf{10^4\ H}$

Also, $\quad \mathcal{E} = N\dfrac{d\phi}{dt} = 200 \times \dfrac{500}{0.5} = \mathbf{2 \times 10^5\ V.}$

VERY SHORT ANSWER CONCEPTUAL PROBLEMS

Problem 1. **What is the basic cause of induced emf ?** [Punjab 01]

Solution. The basic cause of induced emf is the change of magnetic flux linked with a circuit.

Problem 2. **Will an induced emf develop in a conductor, when moved in a direction parallel to the magnetic field ?**

Solution. No, because the magnetic flux linked with the conductor does not change when it is moved parallel to the magnetic field. Moreover, the magnetic Lorentz force on the free electrons of the conductor is zero, so no emf is induced across the ends of the conductor.

Problem 3. **A train is moving with uniform speed from north to south. (*i*) Will any induced emf appear across the ends of its axle ? (*ii*) Will the answer be affected if the train moves from east to west ?**

Solution. (*i*) Yes, emf will appear because the axle is intercepting the vertical component of the earth's magnetic field. (*ii*) No, here also the axle intercepts the vertical component of the earth's magnetic field, so emf is induced across the ends of the axle.

Problem 4. **Does the change in magnetic flux induce emf or current ?**

Solution. The change in magnetic flux always induces emf. However, the current is induced only when the circuit is closed.

Problem 5. **Induced emf is also called back emf. Why ?**

Solution. It is because induced emf always opposes any change in applied emf.

Problem 6. A wire kept along the north-south direction is allowed to fall freely. Will an emf be induced in the wire ?

Solution. No, because neither horizontal nor vertical component of earth's magnetic field will be intercepted by the falling wire.

Problem 7. **A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends ? Justify your answer.** [CBSE D 13]

Solution. Yes, because the horizontal component of earth's magnetic field is intercepted by the rod.

Problem 8. **A cylindrical bar magnet is kept along the axis of a circular coil. Will there be a current induced in the coil if the magnet is rotated about its axis ? Give reason.**

Solution. No, because the magnetic flux linked with the circular coil does not change when the magnet is rotated about its axis.

Problem 9. **A vertical metallic pole falls down through the plane of the magnetic meridian. Will any emf be produced between its ends ? Give reason for your answer.** [CBSE D 95 C]

Solution. No emf will be induced because the metallic pole neither intercepts the horizontal component B_H nor the vertical component B_V of earth's magnetic field.

Problem 10. **The electric current flowing in a wire in the direction *B* to *A* is decreasing. What is the direction of induced current in the metallic loop kept above the wire as shown in Fig. 6.35.** [CBSE OD 14]

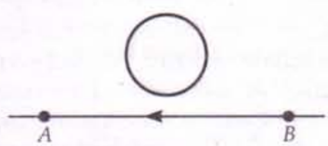


Fig. 6.35

GUIDELINES TO NCERT EXERCISES

6.1. *Predict the direction of induced current in the situations described by the following Figs. 6.99 (a) to (f).*

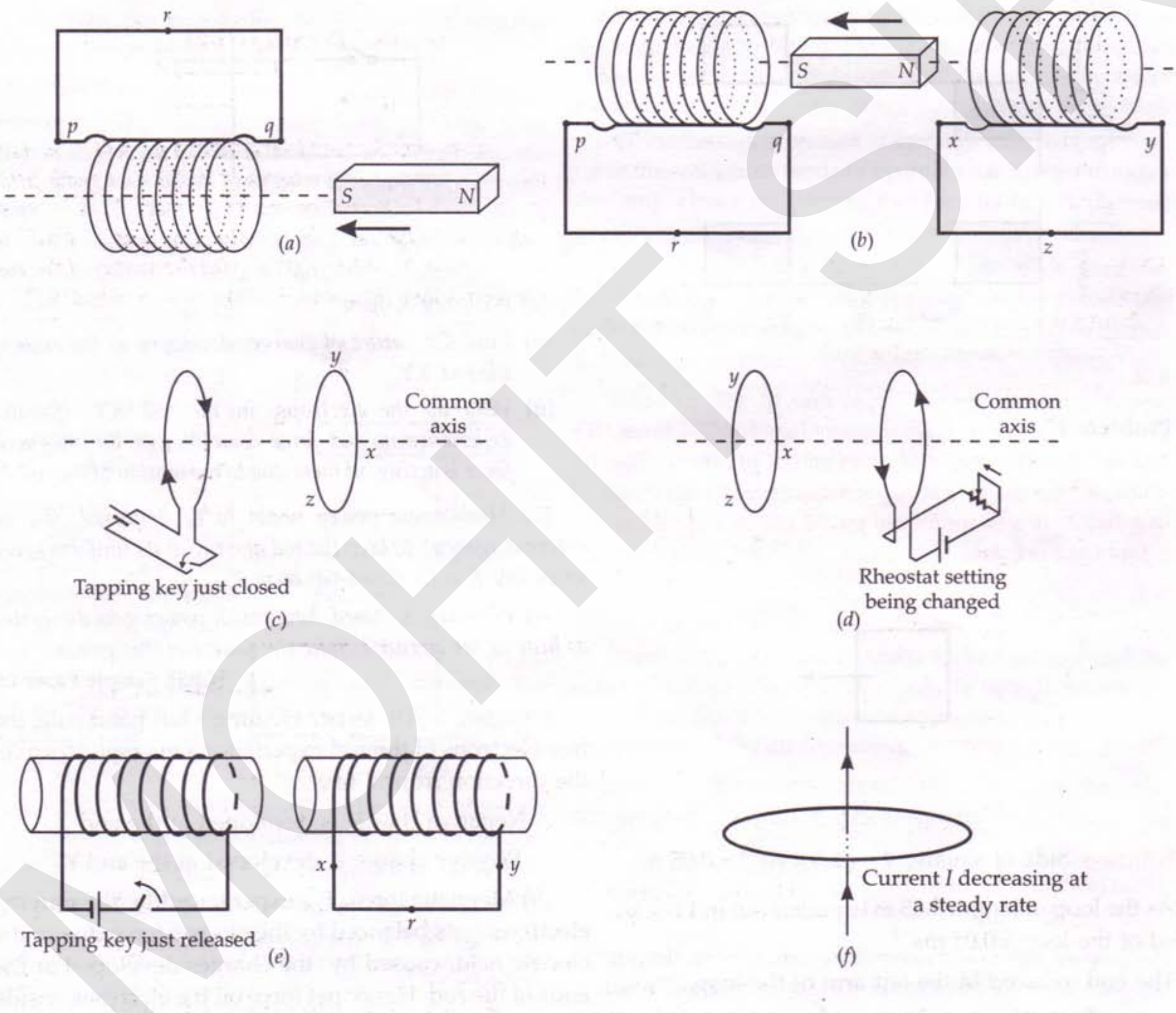


Fig. 6.99

Ans. (*a*) As the S-pole of the magnet approaches the coil, by Lenz's law, the induced current in the coil develops S-pole at the end *q*. For this the induced current should flow clockwise when seen from the magnet side. Hence the induced current flows along *qrpq*.

(*b*) As the magnet is moved, its S-pole moves towards *pq* coil and its N-pole moves away from *xy* coil. By Lenz's law the induced current should develop S-pole at the end *q* and also a S-pole at the end *x*. For this the current in the two coils should flow clockwise when seen from the magnet side. Hence the induced current flows along *prq* in one coil and along *yzx* in the other coil.

(*c*) As the circuit of the left loop is completed, current flows in the direction of arrows shown. This current develops south polarity on the side of right loop. The induced current should flow clockwise (when seen from left) in the right loop so as to oppose the growth of current in left loop. Hence the induced current flows along *yzx*.

(*d*) As the rheostat is adjusted, the resistance decreases and current increases. This increases the flux through the neighbouring coil. By Lenz's rule, increase of flux should be opposed. The induced current should produce flux in the opposite direction of original flux. Hence the induced current flows along *zyx*.

(*e*) As the circuit breaks, the flux decreases. The induced current should flow along *xry* to supplement the flux.

(*f*) The circular field lines set up around the current carrying wire lie in the plane of the loop. The flux threading the coil in the perpendicular direction is zero. Any change in current will not change this flux. Hence no induced current is set up in the coil.

6.2. *Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.100.*

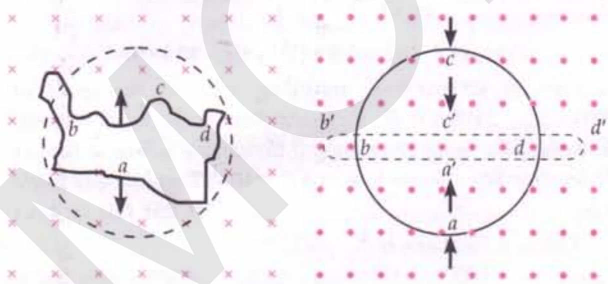


Fig. 6.100

(*a*) *A wire of irregular shape turning into a circular shape.*

(*b*) *A circular loop being deformed into a narrow straight wire.*

Ans. (*a*) As the loop changes from irregular to circular shape, its area increases. Hence the magnetic flux linked with it increases. According to Lenz's law, the induced current should produce magnetic flux in the opposite direction of original flux. For this induced current should flow in the anticlockwise direction, *i.e.*, along *adcb*.

(*b*) As the circular loop is transformed into a narrow wire, its area decreases. The magnetic flux linked with it also decreases. By Lenz's law, the induced current should produce a flux in the direction of original flux. For this the induced current should flow in the anticlockwise direction, *i.e.*, along *a'd'c'b'*.

6.3. *A long solenoid with 15 turns per cm has a small loop of area 2.0 cm² placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop while the current is changing ?*

Ans. Here $n = 15$ turns/cm $= 1500$ turns/m,

$$A = 2.0\ \text{cm}^2 = 2 \times 10^{-4}\ \text{m}^2$$

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A \cdot \frac{dB}{dt} = A \cdot \frac{d}{dt}(\mu_0 nI)$$

$$= \mu_0 nA \frac{dI}{dt}$$

$$= 4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4} \times \left(\frac{4-2}{0.1}\right)$$

$$= \mathbf{7.5 \times 10^{-6}\ V}.$$

6.4. *A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cms⁻¹ in a direction normal to the (i) longer side (ii) shorter side of the loop ? For how long does the induced voltage last in each case ?*

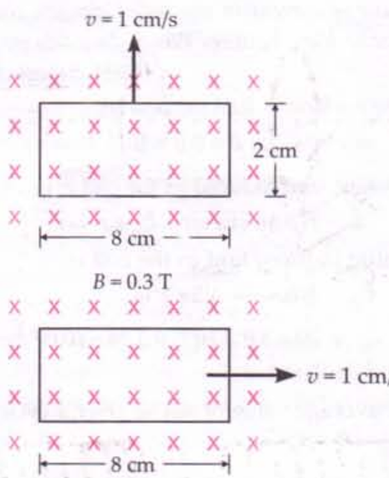


Fig. 6.101

Ans. Here $B = 0.3$ T, $v = 1\ \text{cms}^{-1} = 0.01\ \text{ms}^{-1}$

$l = 8\ \text{cm} = 0.08$ m, $b = 2\ \text{cm} = 0.02$ m

(*i*) *When the loop moves normal to the longer side :*

Induced emf,

$$\varepsilon = Blv = 0.3 \times 0.08 \times 0.01 = \mathbf{2.4 \times 10^{-4}\ V}$$

∴ Time for which emf lasts

= Time in which shorter side moves out of the field

$= \frac{b}{v} = \frac{0.02}{0.01} = \mathbf{2\ s}$

(ii) When the loop moves normal to the shorter side :

Induced emf,

$\varepsilon = Bbv = 0.3 \times 0.02 \times 0.01 = \mathbf{0.6 \times 10^{-4}\ V}$

∴ Time for which emf lasts

= Time in which the longer side moves out of the field

$= \frac{l}{v} = \frac{0.08}{0.01} = \mathbf{8\ s.}$

6.5. *A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.*

Ans. Here $L = 1.0$ m, $\omega = 400$ rad s^{-1}, $B = 0.5$ T

The emf developed between the centre and the ring is

$$\varepsilon = \frac{1}{2} BL^2\omega = \frac{1}{2} \times 0.5 \times (1.0)^2 \times 400 = \mathbf{100\ V.}$$

6.6. *A circular coil of radius 8.0 cm and 20 turns rotates about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and the average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from ?* **[CBSE Sample Paper 08]**

Ans. Here $r = 8$ cm $= 0.08$ m, $N = 20$,

$\omega = 50\ s^{-1}$, $B = 3.0 \times 10^{-2}$ T

At any instant, emf induced in the coil is given by

$$\varepsilon = NBA\omega \sin \omega t = \varepsilon_0 \sin \omega t$$

∴ Maximum induced emf in the coil is

$$\varepsilon_0 = NBA\omega = NB\pi r^2\omega$$
$$= 20 \times 3.0 \times 10^{-2} \times 3.14 \times (0.08)^2 \times 50\ \text{V}$$
$$= \mathbf{0.603\ V}$$

Since the average value of sin ωt over a cycle is zero, therefore, $\varepsilon_{av} = 0$

Maximum induced current, $I_0 = \frac{\varepsilon_0}{R} = \frac{0.603}{10} = \mathbf{0.0603\ A}$

∴ Power dissipated as heat

$$= \frac{1}{2} \cdot \varepsilon_0 I_0 = \frac{1}{2}\frac{\varepsilon_0^2}{R} = \frac{1}{2} \times \frac{(0.603)^2}{10}\ \text{W} = \mathbf{0.018\ W}$$

The source of this power is the external agent which keeps the coil rotating.

6.7. *A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1} at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.*

(a) What is the instantaneous value of the emf induced in the wire ?

(b) What is the direction of the emf ?

(c) Which end of the wire is at the higher electrical potential ?

Ans. (a) Here, $l = 10$ m, $v = 5.0$ ms^{-1},

$B_H = 0.30 \times 10^{-4}$ Wbm^{-2}

$$\varepsilon = B_H lv = 0.30 \times 10^{-4} \times 10 \times 5.0 = \mathbf{1.5 \times 10^{-3}\ V.}$$

(b) According to Fleming's right hand rule, the direction of emf is from ***west*** to ***east***.

(c) ***Western end*** of the wire is at the higher electrical potential.

6.8. *Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.*

Ans. Here, $I_1 = 5.0$ A, $I_2 = 0.0$ A, $t = 0.1$ s, $\varepsilon = 200$ V

As $\varepsilon = -L\frac{dI}{dt} = -L\frac{I_2 - I_1}{t}$

∴ $200 = -L\frac{0.0 - 5.0}{0.1} = +50\ L$

or $L = \mathbf{4\ H.}$

6.9. *A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil ?*

Ans. Here, $M = 1.5$ H, $I_1 = 0$ A, $I_2 = 20$ A, $t = 0.5$ s

As $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

∴ $d\phi = LdI = 1.5 \times (20 - 0) = \mathbf{30\ Wb.}$

6.10. *A jet plane is travelling west at the speed of 1800 km h^{-1}. What is the voltage difference developed between the ends of the wing 25 m long, if the earth's magnetic field at the location has a magnitude of 5.0×10^{-4} T and the dip angle is 30° ?* **[CBSE OD 09, 15C]**

Ans. $v = 1800$ km h^{-1}

$$= \frac{1800 \times 1000}{3600}\ \text{ms}^{-1} = 500\ \text{ms}^{-1}$$

$l = 25$ m, $B = 5.0 \times 10^{-4}$ T, $\delta = 30°$

$$B_V = B \sin \delta = 5.0 \times 10^{-4} \times \sin 30°$$
$$= 2.5 \times 10^{-4}\ \text{T}$$

Only the flux-lines of vertical component of field $\vec{B}$ are cutting across the horizontally moving jet plane.

∴ Induced emf,

$$\varepsilon = B_V lv = 2.5 \times 10^{-4} \times 25 \times 500 \simeq \mathbf{3.125\ V.}$$

6.11. *Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of 0.02 Ts^{-1}. If the cut is joined and the loop has a resistance of 1.6 Ω, how much power is dissipated by the loop as heat ? What is the source of this power ?*

Ans. $A = 8 \times 2 = 16\ cm^2 = 16 \times 10^{-4}\ m^2$,

$$\frac{dB}{dt} = 0.02\ Ts^{-1}$$

$$\therefore \quad \mathcal{E} = \frac{d\phi}{dt} = A\frac{dB}{dt} = 16 \times 10^{-4} \times 0.02 = 3.2 \times 10^{-5}\ V$$

$R = 1.6\ \Omega$

Induced current,

$$I = \frac{\mathcal{E}}{R} = \frac{3.2 \times 10^{-5}}{1.6} = 2 \times 10^{-5}\ A$$

Power dissipated as heat,

$$P = I^2R = (2 \times 10^{-5})^2 \times 1.6 = \mathbf{6.4 \times 10^{-10}\ W.}$$

The source of this power is the external agency responsible for changing the magnetic field with time.

6.12. *A square loop of side 12 cm with its sides parallel to x- and y-axes moves with a velocity of 8 cms^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction (i.e., it increases by 10^{-3} T per cm as one moves in the – ve x-direction), and it is decreasing in time at the rate of $10^{-3}\ Ts^{-1}$. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.5 m Ω.*

Ans. Here $A = a^2 = (0.12\ m)^2 = 144 \times 10^{-4}\ m^2$

$$v = 8\ cm\ s^{-1} = 0.08\ ms^{-1}$$

Rate of change of magnetic field B with distance is

$$\frac{dB}{dx} = 10^{-3}\ T\ cm^{-1} = \frac{10^{-3}\ T}{10^{-2}\ m} = 10^{-1}\ Tm^{-1}$$

Rate of change of magnetic field B with time t is

$$\frac{dB}{dt} = 10^{-3}\ Ts^{-1}$$

Induced emf due to change in field B with position x is given by

$$\mathcal{E}_1 = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A.\frac{dB}{dt} = A.\frac{dx}{dt}.\frac{dB}{dx} = A.v.\frac{dB}{dx}$$

$$= 144 \times 10^{-4} \times 0.08 \times 10^{-1}\ V = 115.2 \times 10^{-6}\ V$$

Induced emf due to change in field B with time t is

$$\mathcal{E}_2 = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt}$$

$$= 144 \times 10^{-4} \times 10^{-3}\ V = 14.4 \times 10^{-6}\ V$$

∴ Total induced emf,

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = (115.2 + 14.4) \times 10^{-6}\ V = 129.6 \times 10^{-6}\ V$$

As $R = 4.5\ m\Omega = 4.5 \times 10^{-3}\ \Omega$

∴ Induced current,

$$I = \frac{\mathcal{E}}{R} = \frac{129.6 \times 10^{-6}}{4.5 \times 10^{-3}}\ A = \mathbf{2.9 \times 10^{-2}\ A}$$

The two effects have been added up because both cause a decrease in flux in +ve *z-direction. The direction of induced current is such as to increase the flux through the loop along* +ve z-direction. If for the observer the loop moves to the right, the current will be seen to be anticlockwise.

6.13. *It is desired to measure the magnitude of field between the poles of a powerful loudspeaker magnet. A small flat search coil of area 2.0 cm^2 with 25 closely wound turns is positioned normal to the field direction and then quickly snatched out of the field region. (Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to the coil) is 7.5 mC. The resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of the magnet.*

Ans. Here $\phi_1 = BA$ and $\phi_2 = 0$

Induced emf,

$$\mathcal{E} = -N\frac{\phi_2 - \phi_1}{t} = -N.\frac{0 - BA}{t} = \frac{NBA}{t}$$

or $$IR = \frac{NBA}{t} \quad \text{or} \quad \frac{q}{t}.R = \frac{NBA}{t}$$

or $$B = \frac{qR}{NA}$$

But $q = 7.5\ mC = 7.5 \times 10^{-3} C$,

$A = 2.0\ cm^2 = 2.0 \times 10^{-4}\ m^2$,

$N = 25,\ R = 0.50\ \Omega$

$$\therefore \quad B = \frac{7.5 \times 10^{-3} \times 0.50}{25 \times 2.0 \times 10^{-4}} = \mathbf{0.75\ Wb\ m^{-2}.}$$

6.14. *Figure 6.102 shows a metal rod PQ resting on the rails AB and positioned between the poles of a permanent magnet. The rails, the rod and the magnetic field are in three mutually perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T,*

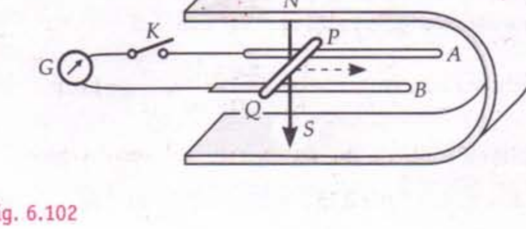


Fig. 6.102

resistance of the closed loop containing the rod = 9.0 m Ω. Assume, the field to be uniform.

(*a*) *Suppose K is open and the rod moves with a speed of 12 cms⁻¹ in the direction shown. Give the polarity and magnitude of the induced emf.*

(*b*) *Is there an excess charge built up at the ends of the rods when K is open ? What if K is closed ?*

(*c*) *With K open and the rod moving uniformly, there is no net force on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.*

(*d*) *What is the retarding force on the rod when K is closed ?*

(*e*) *How much power is required (by an external agent) to keep the rod moving at the same speed (= 12 cms⁻¹) when K is closed ? How much power is required when K is open ?*

(*f*) *How much power is dissipated as heat in the closed circuit ? What is the source of the power ?*

(*g*) *What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular ?*

Ans. (*a*) The magnitude of the induced emf is given by

$$\mathcal{E} = Blv \sin\theta$$

where θ is the angle made by the rod *PQ* with the field $\vec{B}$. In this case θ = 90° so that

$$\mathcal{E} = Blv = 0.50 \times 0.15 \times 0.12 \text{ V}$$
$$= 9.0 \times 10^{-3} \text{ V} = \mathbf{9.0 \text{ mV}}$$

As the conductor *PQ* moves in the direction shown, the free electrons in it experience magnetic Lorentz force. By Fleming's left hand rule, the electrons move from the end *P* towards the end *Q*. Deficiency of electrons makes the end *P* positive while the excess of electrons makes the end *Q* negative.

(*b*) Yes, the excess charge is built up at the ends of the rod because induced emf is set up across the ends as it moves in the field even when the key *K* is open.

When the key *K* is closed, the excess charge is maintained by the continuous flow of current.

(*c*) The magnetic Lorentz force $[\vec{F}_m = -e(\vec{v} \times \vec{B})]$ is cancelled by the electric force $[\vec{F}_e = e\vec{E}]$ exerted by the electric field set up by the opposite charges at its ends.

(*d*) Resistance, $R = 9.0 \text{ m}\Omega = 9.0 \times 10^{-3}\ \Omega$

$$\therefore \text{ Induced current, } I = \frac{\mathcal{E}}{R} = \frac{9.0 \times 10^{-3} \text{ V}}{9.0 \times 10^{-3}\ \Omega} = 1.0 \text{ A}$$

Retarding force on the rod is

$$F = IlB = 1.0 \times 0.15 \times 0.50 \text{ N} = \mathbf{75 \times 10^{-3} \text{ N}}.$$

(*e*) Power expended by an external agent against the retarding force *F* to keep the rod moving uniformly at 12cms⁻¹ is given by

$$P = \text{Force} \times \text{velocity} = Fv$$
$$= 75 \times 10^{-3} \times 12 \times 10^{-2} \text{ W} = \mathbf{9.0 \times 10^{-3} \text{ W}}.$$

(*f*) Power dissipated as heat

$$= I^2R = (1.0)^2 \times 9.0 \times 10^{-3} \text{ W} = \mathbf{9.0 \times 10^{-3} \text{ W}}$$

The source of this power is the external agent which keeps the rod moving against the retarding force *F*.

(*g*) In this case, the angle θ made by the rod with the field $\vec{B}$ is zero.

$$\therefore \quad \mathcal{E} = Blv \sin 0° = 0$$

This is because the motion of the loop does not cut across the field lines. There is no change in magnetic flux. So the induced emf is zero.

6.15. *An air-cored solenoid with length 30 cm, area of cross-section 25 cm² and number of turns 500 carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10⁻³ s. How much is the average back emf induced across the ends of the open switch in the circuit ? Ignore the variation in magnetic field near the ends of the solenoid.*

Ans. The magnetic field inside a solenoid of *N* turns, length *l* and carrying current *I* is

$$B = \frac{\mu_0 NI}{l}$$

Flux linked with *N* turns of the solenoid is

$$\phi = NBA = N\frac{\mu_0 NI}{l} \cdot A = \frac{\mu_0 AN^2 I}{l}$$

Here $l = 30 \text{ cm} = 0.30 \text{ m}$, $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$

$I = 2.5$ *A*, $N = 500$, $\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$

Initial flux linked with the solenoid is

$$\phi_i = \frac{4\pi \times 10^{-7} \times 25 \times 10^{-4} \times (500)^2 \times 2.5}{0.30} \text{ Wb}$$
$$= 6.54 \times 10^{-3} \text{ Wb}$$

Final flux linked with the solenoid (when the current is switched off) is

$$\phi_f = 0$$

Average back emf is

$$\mathcal{E}_{av} = \frac{\text{Total change in flux}}{\text{Total time}} = \frac{6.54 \times 10^{-3} - 0}{10^{-3}} = \mathbf{6.54 \text{ V}}.$$

6.16. (*a*) *Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.103.*

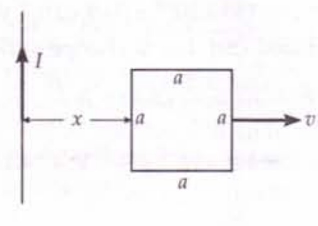


Fig. 6.103

(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, $v = 10$ m/s. Calculate the induced emf in the loop at the instant when $x = 0.2$ m. Take $a = 0.1$ m and assume that the loop has a large resistance.

Ans. (a) As shown in Fig. 6.104, consider a rectangular strip of small width dr of the loop at distance r from the wire.

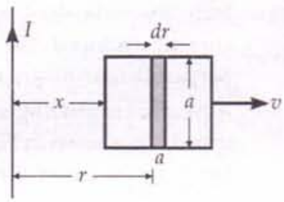


Fig. 6.104

Magnetic field at the location of the strip is

$$B = \frac{\mu_0 I}{2\pi r}$$

This field points normally into the plane of the loop.

Area of the strip, $A = a\,dr$

Magnetic flux linked with the strip,

$$d\phi = BA = \frac{\mu_0 I}{2\pi r} \cdot a\,dr$$

Total magnetic flux linked with the square loop,

$$\phi = \int d\phi = \int_{r=x}^{r=x+a} \frac{\mu_0 I}{2\pi r} a\,dr = \frac{\mu_0 Ia}{2\pi} \int_{r=x}^{r=x+a} \frac{1}{r}\,dr$$

$$= \frac{\mu_0 Ia}{2\pi} [\ln r]_x^{x+a} = \frac{\mu_0 Ia}{2\pi} [\ln (x+a) - \ln x]$$

$$= \frac{\mu_0 Ia}{2\pi} \ln \frac{x+a}{x} = \frac{\mu_0 Ia}{2\pi} \ln \left(1 + \frac{a}{x}\right).$$

(b) The square loop is moving in a non-uniform magnetic field. The magnetic flux linked with the loop at any instant is

$$\phi = \frac{\mu_0 Ia}{2\pi} \ln \left(1 + \frac{a}{x}\right)$$

Induced emf set up in the loop,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d\phi}{dx} \cdot \frac{dx}{dt} = -v\frac{d\phi}{dx}$$

$$= -v\frac{d}{dx}\left[\frac{\mu_0 Ia}{2\pi} \ln \left(1 + \frac{a}{x}\right)\right]$$

$$= -v \cdot \frac{\mu_0 Ia}{2\pi} \cdot \frac{1}{\left(1 + \frac{a}{x}\right)} \cdot \left(-\frac{a}{x^2}\right) = \frac{\mu_0}{2\pi} \frac{a^2 v}{x(x+a)} \cdot I$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{(0.1)^2 \times 10}{0.2(0.2 + 0.1)} \times 50$$

$$= 1.67 \times 10^{-5}\,\text{V} \simeq \mathbf{1.7 \times 10^{-5}\,V}.$$

6.17. *A line charge λ per unit length is lodged uniformly onto the wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.105). A uniform magnetic field extends over a circular region within the rim.*

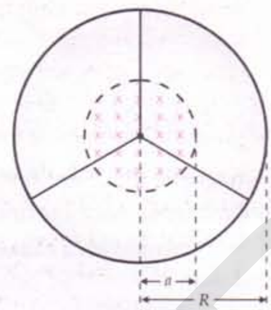


Fig. 6.105

It is given by

$$\vec{B} = -B_0\,\hat{k} \quad [r \le a, a < R]$$

$$= 0 \quad \text{(otherwise)}$$

What is the angular velocity of the wheel after the field is suddenly switched off ?

Ans. According to Faraday's law of electromagnetic induction, the induced emf is

$$\varepsilon = -\frac{d\phi}{dt}$$

This implies the presence of an electric field $\vec{E}$ tangent to all points of the boundary of the circular region of radius a. If we move a test charge q once round this boundary, a work $q\varepsilon$ will be done. The electric force on the charge is qE and work done by this force round this boundary is $qE \times 2\pi a$. Equating the two works done, we get

$$q\varepsilon = qE \times 2\pi a \quad \text{or} \quad E = \frac{\varepsilon}{2\pi a}$$

As $\varepsilon = -\dfrac{d\phi}{dt}$,

so $$E = -\frac{1}{2\pi a} \cdot \frac{d\phi}{dt} = -\frac{1}{2\pi a} \cdot \frac{d}{dt}(\pi a^2 B) = -\frac{a}{2}\frac{dB}{dt}$$

In the given problem, total charge on the rim = $\lambda \times 2\pi a$. Therefore, the force on this charge is

$$F = \lambda \cdot 2\pi a \cdot E = -\lambda \cdot 2\pi a \cdot \frac{a}{2} \cdot \frac{dB}{dt}$$

or $$m \cdot \frac{dv}{dt} = -\lambda \cdot 2\pi a \cdot \frac{a}{2}\frac{dB}{dt} \qquad \left[\because F = ma = m\frac{dv}{dt}\right]$$

or $$m \cdot \frac{d}{dt}(R\omega) = -\lambda \pi a^2 \frac{dB}{dt}$$

or $$mR\frac{d\omega}{dt} = -\lambda \pi a^2 \frac{dB}{dt}$$

or $$d\omega = -\frac{\lambda \pi a^2}{mR} dB$$

Integrating both sides, $\omega = -\dfrac{\lambda \pi a^2 B}{mR}$

In vector notation, $\vec{\omega} = -\dfrac{B\pi a^2 \lambda}{mR}\hat{k}$

The negative sign indicates that the vector $\vec{\omega}$ is in the negative z-direction.

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. When is the magnetic flux crossing a given surface area held in a magnetic field maximum ?
2. When is the flux linked with a closed coil held in a magnetic field zero ?
3. Name the SI units of (*i*) magnetic flux and (*ii*) magnetic induction (or magnetic flux density). [Himachal 93]
4. Write the dimensional formula of magnetic flux. [Punjab 99C]
5. Define weber. How is it related to Maxwell ?
6. State Faraday's law of electromagnetic induction. [CBSE OD 09 ; F 09]
7. A coil of metal wire is held stationary in a non-uniform magnetic field. Is any emf induced in the coil ?
8. On what factors does the magnitude of the emf induced in the circuit due to magnetic flux depend ? [CBSE F 13]
9. A wire cuts across a flux of 0.2×10^{-2} Wb in 0.12 s. What is the emf induced in the wire ? [ICSE 96]
10. A metallic wire 1 m in length is moving normally across a field of 0.1 T with a speed of 5 ms^{-1}. Find the emf between the ends of the wire. [ISCE 97]
11. How would you detect the presence of magnetic field on an unknown planet ?
12. When current flowing in an inductive circuit is switched off, will the induced current be in the direction of main current or in opposite direction ?
13. Give the expression for the emf induced between the ends of a metal conductor moving perpendicular to a uniform magnetic field. [CBSE D 93C]
14. A glass rod of length *l* moves with velocity *v* perpendicular to a uniform magnetic field *B*. What is the induced emf in the rod ?
15. A coil of area *A* is kept perpendicular in a uniform magnetic field *B*. If the coil is rotated by 180°, what will be the change in flux ?
16. A bicycle generator creates a 3.0 V, when the bicycle is travelling at a speed of 9.0 kmh^{-1}. How much emf is generated when the bicycle is travelling at a 15 kmh^{-1} ?
17. State the law that gives the polarity of the induced emf. [CBSE OD 09]

Or

State Lenz's law. [CBSE D 13]

18. State the rule used to determine the direction of current induced in a conductor moving in a perpendicular magnetic field.
19. A magnet is moving towards a coil with a uniform speed *v* as shown in Fig. 6.106. State the direction of the induced current in the resistor *R*. [CBSE Sample Paper 13]

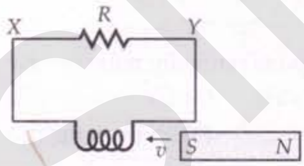


Fig. 6.106

20. A conducting loop is held below a current carrying wire *PQ* as shown in Fig. 6.107. Predict the direction of the induced current in the loop when the current in the wire is constantly increasing. [CBSE OD 14]

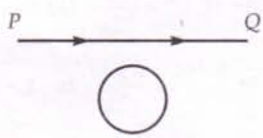


Fig. 6.107

21. Predict the direction of induced current in metal rings 1 and 2 when current *I* in the wire is steadily decreasing. (Fig. 6.108) [CBSE D 12]

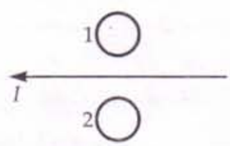


Fig. 6.108

22. Predict the direction of induced current in a metal ring when the ring is moved towards a straight conductor with constant speed *v*. The conductor is carrying current *I* in the direction shown in Fig. 6.109 [CBSE D 12]

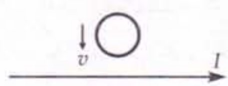


Fig. 6.109

23. Define the unit of self-inductance. [Haryana 98C ; Punjab 2000]

24. If the number of turns and the length of the solenoid are doubled keeping the area of cross-section same, how is the inductance affected ? [CBSE.PMT 93]
25. Two coils of self inductance L_1 and L_2 are connected in series, so that current flows in the same sense and they have a mutual inductance M. What is their equivalent inductance ?
26. If the number of turns of a solenoid is doubled, keeping the other factors constant, how does the self-inductance of the solenoid change ? [CBSE D 2000]
27. A coil of wire of certain radius has 600 turns and a self-inductance of 108 mH. What will be the self-inductance of another similar coil with 500 turns ?
28. Write three factors on which the self-inductance of coil depends. [CBSE OD 95]
29. What is meant by mutual induction ? [Himachal 98]
30. Define coefficient of mutual inductance for a pair of coils. [ISCE 98]
31. What is the unit of mutual inductance. [ISCE 96]
32. Define mutual inductance of one henry.
33. Two circular loops are placed with their centres separated by a fixed distance. How would you orient the loops to have (*i*) the largest mutual inductance and (*ii*) the smallest mutual inductance ?
34. Find the dimensions of mutual inductance.
35. What will be the dimensions of L/R, if L is inductance and R is resistance ?
36. Show that the SI unit of inductance, henry is equal to volt second per ampere. [ISCE 03]
37. When current in a coil changes with time, how is the back emf induced in the coil related to it ? [CBSE OD 08]
38. Mention any one useful application of eddy currents. [CBSE D 09]
39. Define self-inductance. Give its SI units. [CBSE F 09]
40. Define mutual inductance. Give its SI units. [CBSE F 09]
41. How does the mutual inductance of a pair of coils change, when (*i*) distance between the coils is increased (*ii*) number of turns in each coil is increased ? [CBSE OD 13]
42. Figure 6.110 shows a current carrying solenoid moving towards a conducting loop. Find the direction of the current induced in the loop. [CBSE D 15C]

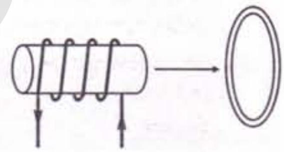


Fig. 6.110

Answers

1. The flux is maximum when the area is held perpendicular to the direction of the magnetic field.
2. When the plane of the coil is parallel to the magnetic field, flux linked with it is zero.
3. (*i*) SI unit of magnetic flux is *weber* (Wb)
   (*ii*) SI unit of magnetic induction is *tesla* (T).
4. $\phi = BA = \dfrac{FA}{qv \sin \theta}$

   $\therefore \quad [\phi] = \dfrac{MLT^{-2} . L^2}{AT . LT^{-1} . 1} = [ML^2T^{-2}A^{-1}]$.
5. One weber is the flux produced when a uniform magnetic field of one tesla acts normally over an area of 1 m^2.

   1 weber = 10^8 maxwell.
6. Refer to point 4 of Glimpses.
7. No emf is induced because the magnetic flux linked with the stationary coil is not changing.
8. The magnitude of induced emf is directly proportional to the rate of change of magnetic flux linked with the circuit.
9. $|\varepsilon| = \dfrac{d\phi}{dt} = \dfrac{0.2 \times 10^{-2}}{0.12} = \mathbf{0.0167\ V}$.
10. $\varepsilon = Blv = 0.1 \times 1 \times 5 = \mathbf{0.5\ V}$.
11. Take closed coil connected to a sensitive galvanometer and rotate the coil. If the galvanometer shows a deflection, a magnetic field is present on the planet, otherwise not.
12. When the circuit is switched off, induced current is in the same direction as the main current.
13. $\varepsilon = \boldsymbol{Blv}$.
14. Zero, because glass rod is an insulator.
15. Change in flux

    $= \phi_2 - \phi_1 = BA \cos 180° - BA \cos 0° = \mathbf{-2\,BA}$.
16. As $\quad \varepsilon = Blv$

    $\therefore \quad \varepsilon \propto v \quad$ or $\quad \dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{v_1}{v_2}$

    $\therefore \quad \dfrac{3.0}{\varepsilon_2} = \dfrac{9.0}{15} \quad$ or $\quad \varepsilon_2 = \mathbf{5.0\ V}$.
17. Lenz's law states that the direction of induced current is such that it opposes the cause which produces it.
18. Fleming's right hand rule can be used to find the direction of current induced in a conductor. Stretch the thumb and the first two fingers of the

right hand in mutually perpendicular directions. If the first finger points in the direction of the magnetic field, thumb in the direction of motion of the conductor, then the central finger will point in the direction of current induced in the conductor.

19. As per Lenz's law, the induced current flows from X to Y through the resistor R.
20. Anticlockwise.
21. Clockwise in ring 1 and anticlockwise in ring 2.
22. Induced current flows clockwise in the metal ring.
23. The SI unit of self-inductance is henry (H). The self-inductance of a coil is said to be one henry if an induced emf of one volt is set up in it when the current in it changes at the rate of one ampere per second.
24. Inductance gets doubled, because
$$L = \frac{\mu_0 N^2 A}{l}.$$
25. $L_{eq} = L_1 + L_2 + 2M$.
26. As $L = \dfrac{\mu_0 N^2 . A}{l}$,

so when N is doubled, the self- inductance becomes four times the original one.

27. As $L \propto N^2$
$$\therefore \quad \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}$$
$$L_2 = \frac{(500)^2}{(600)^2} \times 108 = \mathbf{75\ mH}.$$
28. The self-inductance of a coil depends on
    (*i*) its number of turns,
    (*ii*) its area of cross-section, and
    (*iii*) the permeability of the core material.
29. The phenomenon of production of induced emf in one coil due to a change of current in the neighbouring coil is called mutual induction.
30. The mutual inductance of two coils is defined as the induced emf set up in one coil when the current in the neighbouring coil changes at the unit rate.
31. The SI unit of mutual inductance is henry.
32. The mutual inductance of two coils is said to be one henry if an induced emf of 1 volt is set up in one coil when the current in the neighbouring coil changes at the rate of 1 ampere per second.
33. (*i*) When the planes of the two loops are held parallel to each other, mutual inductance is largest.
    (*ii*) When the planes of the loops are held perpendicular to each other, mutual inductance is smallest.
34. $M = \dfrac{\phi}{I} = \dfrac{BA}{I} = \dfrac{F}{qv \sin\theta} \cdot \dfrac{A}{I}$

$\therefore$ Dimensions of M
$$= \frac{MLT^{-2}.L^2}{CLT^{-1}.A} = \frac{ML^2T^{-2}}{A.A} = [ML^2T^{-2}A^{-2}].$$
35. $M^0L^0T^1$.
36. As $L = \dfrac{\varepsilon}{dI/dt}$

$\therefore$ SI unit of $L = \dfrac{1\ V}{1\ As^{-1}}$

or 1 henry (H) = $1\ VsA^{-1}$.

37. Back emf induced $\propto$ Rate of change of current in the coil
$$\varepsilon \propto \frac{dI}{dt} \quad \text{or} \quad \varepsilon = -L\frac{dI}{dt}$$
38. Eddy currents are used in electric furnaces to melt metals.
39. Refer answer to Q.14 on page 6.60.
40. Refer answer to Q.16 on page 6.60.
41. (*i*) M decreases (*ii*) M increases.
42. Clockwise when seen from the solenoid side.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Define magnetic flux. Compute its dimensions.
2. When is the magnetic flux taken as (*i*) positive and (*ii*) negative ?
3. What is electromagnetic induction ? Give an experiment which demonstrates this phenomenon.
4. State Faraday's laws of electro magnetic induction. Express them mathematically. [CBSE OD 95C, 99]
5. State Lenz's law. Give one example to illustrate this law. The "Lenz's law is a consequence of the law of conservation of energy". Justify this statement. [CBSE D 09]

6. Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it. [CBSE D 14]
7. State Lenz's law. Prove that the charge induced is independent of time. [CBSE D 92C ; Punjab 95]
8. Prove that the magnitude of the emf induced in a conductor of length l when it moves at v m/s perpendicular to a uniform magnetic field B is Blv. [ISCE 03 ; CBSE D 06]
9. Derive an expression for the induced emf produced by changing the area of a rectangular coil placed perpendicular to a magnetic field. [CBSE D 92]
10. A rectangular coil of N turns, area A is held in a uniform magnetic field B. If the coil is rotated at a steady angular speed ω, deduce an expression for the induced emf in the coil at any instant of time.

Or [CBSE 99]

A coil of number of turns N, area A is rotated at a constant angular speed ω in a uniform magnetic field B, and connected to a resistor R. Deduce expressions for : (*i*) Maximum emf induced in the coil (*ii*) Power dissipation in the coil. [CBSE 06C, 08]
11. What are eddy currents ? How are these minimised ? Mention two applications of eddy currents. [CBSE OD 06 C, 09]
12. Define the term 'eddy currents'. State the main undesirable effect of these currents and give the method used to minimise this undesirable effect. Write any two applications of eddy currents. [CBSE OD 06 ; D 07C]
13. What is electromagnetic damping ? How is a galvanometer made dead beat ?
14. Define the term self-inductance. Write its SI unit. Give two factors on which self inductance of an air core coil depends. [CBSE OD 15]
15. Define the term 'self-inductance'. Give its unit. Write an expression for the energy stored in an inductor when a steady current 'I' is passed through it. Is this energy electric or magnetic ? [CBSE OD 04C]
16. Define mutual inductance. Write its SI unit. Give two factors on which the coefficient of mutual inductance between a pair of coils depends. [CBSE D 15, OD 15C]
17. Derive an expression for the mutual inductance of two long solenoids wound over one another, in terms of their number of turns N_1, N_2 ; common cross-sectional area A and common length l. [CBSE OD 90]
18. Derive the expression for the self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length. [CBSE D 12 ; OD 13C]
19. Define self-inductance and give its SI unit. Derive an expression for self-inductance of a long, air-cored solenoid of length l, radius r and having N number of turns. [CBSE OD 05 ; D 09]

Or

Deduce an expression for the self-inductance of a long solenoid of N turns, having a core of relative permeability μ_r. [CBSE D 06C, 08]
20. Define the coefficient of mutual induction.

A long solenoid, of length l and radius r_1, is enclosed coaxially within another long solenoid of length l and radius r_2 ($r_2 > r_1$ and $l > r_2$). Deduce the expression for the mutual inductance of this pair of solenoids. [CBSE OD 05 ; D 09C]
21. Obtain the expression for the mutual inductance of a pair of coaxial circular coils of radii r and R ($R >> r$) placed with their centres coinciding. [CBSE D 08]
22. (*i*) How are eddy currents reduced in a metallic core ?
(*ii*) Give two uses of eddy currents. [CBSE F 09]

Answers

1. Refer answer to Q. 1 on page 6.1.
2. Refer answer to Q. 1 on page 6.1.
3. Refer answer to Q. 3 on page 6.2.
4. Refer answer to Q. 4 on page 6.4.
5. Refer answer to Q. 6 on page 6.5.
6. See illustrations in the answer of Q. 5 on page 6.4.
7. Refer answer to Q. 10 on page 6.10.
8. Refer answer to Q. 11(2) on page 6.15.
9. Refer answer to Q. 11(2) on page 6.15.
10. Refer answer to Q. 11(3) on page 6.15.

Instantaneous power,

$$P = \frac{\varepsilon^2}{R} = \frac{\varepsilon_0^2}{R}\sin^2 \omega t$$

Average power dissipated per cycle,

$$P_{av} = \frac{\varepsilon_0^2}{R} < \sin^2 \omega t > = \frac{\varepsilon_0^2}{R} \cdot \frac{1}{2}$$

$$= \frac{N^2 B^2 A^2 \omega^2}{2R}.$$

11. Eddy currents are the currents induced in solid metallic masses when changing magnetic flux passes through them.

The eddy currents can be reduced by using laminated iron cores which consist of thin iron sheets insulated from each other. The eddy currents can also be reduced by using slotted iron blocks.

Eddy currents have following applications :

(*i*) They are used to make galvanometers dead beat.

(*ii*) In electric furnaces to melt metals.

12. Eddy currents are the currents induced in solid metallic masses when changing magnetic flux passes through them. Changing magnetic fields set up current loops in the metallic masses. These loops are irregularly shaped.

Eddy currents are considered undesirable in a transformer because they dissipate energy in the form of heat. To reduce the loss, the core is made of a large number of thin layers separated by insulating material. This increases the resistance of the possible paths and hence reduces the eddy currents. For applications of eddy currents, refer answer to the above question.

13. Refer answer to Q. 12 on page 6.19.

14. The self-inductance of a coil is defined as the induced emf set up in the coil when the current through it changes at the unit rate.

SI unit of self-inductance is henry (H).

The self-inductance of an air-core depends on

(*i*) the number of turns in the coil, and

(*ii*) the area of cross-section of the coil.

15. Refer answer to above question.

When a current I flows through an inductor of inductance L, the energy stored in it is

$$U = \frac{1}{2} LI^2$$

This energy is stored as magnetic energy.

16. **Mutual inductance.** The mutual inductance of two coils is defined as the induced emf set up in one coil when the current in the neighboring coil changes at the unit rate.

SI unit of mutual inductance is henry (H).

The mutual inductance of two coils depends on

(*i*) the number of turns and the geometrical shape of the two coils.

(*ii*) the relative orientation of the two coils.

17. Refer answer to Q. 18 on page 6.23.

18. Refer answer to Q. 15 on page 6.21.

19. Refer answer to Q. 15 on page 6.21.

20. Refer answer to Q. 18 on page 6.23.

21. Refer to the solution of Problem 29 on page 6.40.

22. Refer answer to Q.12 above.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. State Faraday's laws of electromagnetic induction and explain three methods of producing induced emf. [Punjab 02, 03]

2. (*a*) State the law which relates to generation of induced emf in a conductor being moved in a magnetic field.

(*b*) A rod of length l is moved horizontally with a uniform velocity 'v' in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.

(*c*) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductors ? Explain. [CBSE OD 14 ; SP 15]

3. (*a*) State Faraday's law of electromagnetic induction.

(*b*) A horizontal straight wire of length L extending from east to west is falling with speed v at right angles to the horizontal component of Earth's magnetic field B.

(*i*) Write the expression for the instantaneous value of the emf induced in the wire.

(*ii*) What is the direction of the emf ?

(*iii*) Which end of the wire is at the higher potential ? [CBSE OD 11]

4. What are eddy currents ? How are they produced ? Describe briefly three main useful applications of eddy currents. [CBSE F 15]

5. Derive an expression for the induced emf set up in a coil when it is rotated in a uniform magnetic field with a uniform angular velocity. Explain how does the emf vary when the coil turns through an angle of 2π ? What is the instantaneous value of induced emf when the plane of the coil makes an angle of 60° with the magnetic lines. [CBSE OD 95C]

6. Distinguish between self-induction and mutual induction. Calculate self-inductance of a long solenoid of length l, number of turns N and radius r. [CBSE D 92C]

7. (*a*) Define mutual inductance. Deduce an expression for the mutual inductance of two long coaxial solenoids having different radii and different number of turns.
   (*b*) A coil is mechanically rotated with angular speed ω in a uniform magnetic field which is perpendicular to the axis of rotation of the coil. The plane of the coil is initially perpendicular to the field. Plot a graph showing variation of (*i*) magnetic flux ϕ and (*ii*) the induced emf in the coil as a function of ωt. [CBSE OD 09, F13]

8. (*a*) Define mutual inductance and write its SI units.
   (*b*) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.
   (*c*) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for the emf induced in the coil C_1 due to a change in the current through the coil C_2. [CBSE D 15]

Answers

1. Refer answers to Q.3 on page 6.2 & Q.4 on page 6.4.
2. (*a*) According to Faraday's flux rule, the magnitude of induced emf is equal to the rate of change of magnetic flux linked with the closed circuit.

$$\mathcal{E} = -\frac{d\phi}{dt}$$

   (*b*) Refer answer to Q. 7 on page 6.9.
   (*c*) Refer answer to Q. 9 on page 6.10.
3. (*a*) Refer to point 4 of Glimpses
   (*b*) (*i*) $\mathcal{E} = B_H Lv$
   (*ii*) Induced emf will be in the direction from west to east.
   (*iii*) West end of the wire will be at higher potential.
4. Refer answer to Q.12 on page 6.18.
5. Refer answer to Q.11(3) on page 6.15.
   Instantaneous value of induced emf,

$$\mathcal{E} = \mathcal{E}_0 \sin(90° - 60°)$$
$$= \mathcal{E}_0 \sin 30° = \mathcal{E}_0 / 2$$

6. Refer to solution of Problem 7 on page 6.45. Also, refer answer to Q.15 on page 6.21.
7. (*a*) Refer answer to Q.18 on page 6.23.
   (*b*) The variation of magnetic flux ($\phi = BA\cos\omega t$) and induced emf ($\mathcal{E} = \mathcal{E}_0 \sin\omega t$) as a function of ωt is shown below.

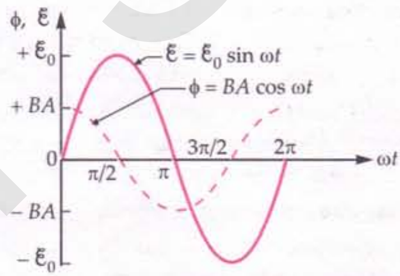


Fig. 6.111

8. (*a*) Refer answer to Q.17 on page 6.22.
   (*b*) Refer answer to Q. 18 on page 6.23.
   (*c*) Flux linked with coil $C_1 \propto$ Current in coil C_2

   or $\phi_1 \propto I_2$ or $\phi_1 = MI_2$

$$\therefore \quad \frac{d\phi_1}{dt} = M\frac{dI_2}{dt}$$

   EMF induced in coil C_1,

$$\mathcal{E} = -\frac{d\phi_1}{dt} = -M\frac{dI_2}{dt}$$

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Vikas once observed that the large number of electric generators used in areas where small workshops existed, produced lot of pollution. He decided to do something for controlling pollution. He alongwith his some friends, made a survey and concluded that like in society apartments, a common generator could be set up for all such workshops, so that noise and air pollution could be reduced considerably. They had a tough time in convincing the local bodies and they were going to

some NGOs and other financiers to help them to set up a big generator. It was admirable to see their perseverance.

(*a*) What are the values being displayed by Vikas and his friends ?

(*b*) State the factors on which the induced emf in a coil rotating in a uniform magnetic field depends.

Answer

1. (*a*) Team spirit, patience, tolerance, magnanimity, determination, responsibility and dutifulness.

(*b*) Induced emf,

$$\mathcal{E} = NBA\omega \sin \omega t$$

Clearly, induced emf set up in the coil depends on

(*i*) number of turns of the coil.

(*ii*) area of the coil.

(*iii*) angular speed of rotation of the coil and

(*iv*) strength of the magnetic field.

COMPETITION SECTION

Electromagnetic Induction

GLIMPSES

1. Magnetic flux. The number of magnetic lines of force crossing a surface normally is called magnetic flux linked with the surface. If the normal drawn to the surface area A makes angle θ with the field $\vec{B}$, then the magnetic flux is

$$\phi = BA\cos\theta = \vec{B}\cdot\vec{A}$$

Magnetic flux is a scalar quantity. Its dimensions are $[ML^2A^{-1}T^{-2}]$.

2. SI unit of magnetic flux is weber (Wb). It is the flux produced when a uniform magnetic field of 1 T crosses normally an area of 1 m^2. The CGS unit of magnetic flux is *maxwell*.

$$1 \text{ weber} = 10^8 \text{ maxwell}$$

3. Electromagnetic induction. It is the phenomenon of production of induced emf (and hence induced current) due to a change of magnetic flux linked with a closed circuit. The term electromagnetic induction means inducing electricity by magnetism.

4. Faraday's laws of electromagnetic induction. ***First law.*** Whenever the magnetic flux linked with a closed circuit changes, an emf is induced in it which lasts only so long as the change in flux is taking place.

Second law. The magnitude of induced emf is equal to the rate of change of magnetic flux linked with the closed circuit. Mathematically,

$$|\varepsilon| = \frac{d\phi}{dt}$$

5. Lenz's law. It states that the direction of induced current is such that it opposes the cause which produces it *i.e.*, it opposes the change in flux.

6. Mathematical form of the laws of electromagnetic induction. For a coil of one turn, induced emf is given by

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{\phi_2 - \phi_1}{t}$$

For a coil of N turns, $\varepsilon = -N\frac{d\phi}{dt} = -N\frac{\phi_2 - \phi_1}{t}$.

The negative sign shows that induced emf opposes the change in flux. It is in accordance with Lenz's law.

7. Motional emf. The emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf. It is produced due to the magnetic Lorentz force acting on the free electrons of the conductor. If a conductor of length l moves with velocity v in a magnetic field B perpendicular to both its length and the direction of the magnetic field, then the emf induced across its ends is given by $\varepsilon = Blv$

Induced current, $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$

Force necessary to move the conductor,

$$F = \frac{B^2l^2v}{R}$$

Power dissipated as Joule heating loss,

$$P = Fv = \frac{B^2l^2v^2}{R}$$

8. Fleming's right hand rule. It gives the direction of induced emf. If we stretch the thumb and the first two fingers of our right hand mutually perpendicular to each other and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor, then the central finger points in the direction of current induced in the conductor.

9. Relation between induced charge and change in magnetic flux. The induced charge flowing through a circuit depends on the net change in magnetic flux linked with circuit and is independent of the time interval of the flux change.

$$\Delta q = \frac{\Delta \phi}{R} = \frac{\text{Net change in magnetic flux}}{\text{Resistance}}$$

10. Methods of generating induced emf. As $\phi = BA \cos \theta$, so the magnetic flux linked with a loop can be changed and hence induced emf can be produced by three methods : (*i*) by changing the magnetic field B, (*ii*) by changing the area A of the loop and (*iii*) by changing the relative orientation θ of the loop and the magnetic field.

11. Motional emf in a rotating coil. If an N turn coil of area A is rotated with uniform angular velocity ω in a uniform magnetic field B about an axis perpendicular to the field B, then the motional emf set up across the ends of the coil is given by

$$\mathcal{E} = NBA\,\omega \sin \omega t = \mathcal{E}_0 \sin \omega t = \mathcal{E}_0 \sin 2\pi\, ft$$

where $\mathcal{E}_0 = NBA\,\omega$ = peak value of induced emf. Both the direction and magnitude of the induced emf change regularly with time. Such a sinusoidally varying emf is called alternating emf.

12. Eddy currents. These are the currents induced in solid metallic masses when the magnetic flux threading through them changes. Such currents flow in the form of irregularly shaped loops throughout the body of the metal and their direction is given by Lenz's law. Eddy currents cause unnecessary heating and wastage of power. They are reduced by using laminated soft iron cores. They are useful in (*i*) electric brakes, (*ii*) speedometers, (*iii*) induction furnaces and (*iv*) electromagnetic shielding.

13. Self induction. It is phenomenon of production of induced emf in a coil when a changing current passes through it.

14. Self-inductance or coefficient of self induction (L). When a current I flows through a coil, flux linked with it is $\phi = LI$

Induced emf, $\mathcal{E} = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

Thus self-inductance of a coil is the induced emf set up in it when the current passing through it changes at the unit rate. It is a measure of the opposition of the coil to the flow of current through it.

15. Self-inductance of a long solenoid. The self-inductance of a long solenoid of length l, area of cross-section A and having N turns is

$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 lA, \quad \text{where } n = \frac{N}{l}$$

When the solenoid is wound over a soft iron core of relative permeability μ_r, $L = \mu_r \mu_0 n^2 lA$

The value of self inductance depends on the number of turns in the solenoid, its area of cross-section and the relative permeability of its core material.

16. Mutual induction. It is the phenomenon of production of induced emf in one coil when the current through the neighbouring coil changes at the unit rate.

17. Mutual-inductance or coefficient of mutual induction (M). If a current I flowing through one coil generates flux ϕ in the neighbouring coil, then $\phi = MI$

Induced emf, $\mathcal{E} = -\frac{d\phi}{dt} = -M\frac{dI}{dt}$

Thus mutual-inductance of two coils may be defined as the induced emf set up in one coil when the current in the neighbouring coil changes at the unit rate.

18. Mutual inductance of two long solenoids. The mutual inductance of two long co-axial solenoids wound over one another is

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 Al = \mu_0 n_1 n_2 \pi r_1^2 l$$

where n_1, n_2 are the number of turns per unit length of the solenoids, l is their common length and $A = \pi r_1^2$ = cross-sectional area of the inner solenoid.

19. Henry. It is SI unit for both self and mutual-inductances. Inductance is one henry if an induced emf of 1 volt is set up when the current changes at the rate of 1 ampere per second.

$$1 \text{ henry (H)} = 1 \text{ VsA}^{-1} = 1 \text{ Wb A}^{-1}.$$

20. Coefficient of coupling. It gives a measure of the manner in which two coils are coupled together. If L_1 and L_2 are the self-inductances of the two coils and M is the mutual inductance, then the coefficient of coupling

$$K = \sqrt{\frac{M}{L_1 L_2}}$$

The value of K lies between 0 and 1.